LOYOLA COLLEGE (AUTONOMOUS),CHENNAI-600034
B.Sc. DEGREE EXAMINATION-MATHEMATICS
SUBCODE: MT 1501 SUB NAME: GRAPHS, DIFFERENCE EQUATIONS
MATRICES AND FOURIER SERIES

Time: 3 hrs   **Max Marks: 100**

Part- A

Answer ALL the questions.   **(10×2=20)**

1. Check whether each of the following define a function (i) $-6x+1$ (ii) $y^2 = x$.

2. Give the domain and range of the function $f(x) = \dfrac{1}{x+3}$.

3. Write the normal equations of $y = ax + b$.

4. Reduce into linear form: $y = ae^{bx}$ where a and b are constants.

5. Solve $16u_{x+2} - 8u_{x+1} + u_x = 0$

6. Find the particular integral of the equation $y_{k+2} - 5y_{k+1} + 6y_k = 6^k$.

7. State Cayley Hamilton theorem.

8. Find the eigen value of the matrix $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.

9. Find the Fourier coefficient a_0 for the function $f(x) = x\sin x, 0 < x < 2\pi$.

10. State the properties of even and odd function.

Part – B

Answer any FIVE questions.   **(5×8=40)**

11. The cost in rupees to produce x kilograms of chocolate candy is given by $C(x) = 3.5x + 800$. Find each of the following.
    a) The fixed cost
    b) The total cost for 12 kilograms.
    c) The marginal cost per kilogram
    d) The average cost per kilogram.

12. a) The price and demand for an item are related by $p = 32 - x^2$, while price and supply are related by $p = x^2$. Find the equilibrium supply and the equilibrium price.

    b) A firm producing poultry feed finds that the total cost $C(x)$ of producing x units is given by $C(x) = 20x + 100$. Management plans to charge Rs.24 per unit for the feed. (i) How many units must be sold for the firm to break even? (ii) What is the profit if 100 units of feed are sold?

13. Use the method of least squares to fit a straight line to the following data:

x	0	5	10	15	20
y	7	11	16	20	26

Estimate the value of y when $x=25$.

14. The weights of a calf taken at weekly intervals are given below. Use methods of group averages to fit a straight line.

Age in weeks	1	2	3	4	5	6	7	8	9	10
Weight	52.5	58.7	65.0	70.2	75.4	81.1	87.2	95.5	102.2	108.4

Calculate also the average rate of growth per week.

15. Solve $y_{n+2} - 6y_{n+1} + 8y_n = 4^n$.

16. Given $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$, express $A^4 - 4A^3 - A^2 + 2A - 5I$ as a linear polynomial in A.

17. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

18. Obtain a Fourier expansion for the function $f(x) = \dfrac{1}{2}(\pi - x), (0 < x < 2\pi)$.

Part – C

Answer any TWO questions.   **(2×20=40)**

19. a) Graph the function $f(x) = 3x^2 - 12x + 14$ by completing the square.

b) When a company sells x units of a product, its profit is $p(x) = 120x - x^2$. Find the number of units that should be sold so that maximum profit is received. What is the maximum profit?

c) Fit a curve of the form $y = a + bx + cx^2$ to the following data taking x as the independent variable.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Find y when $x=2$.   (4+4+12)

20. Solve the difference equations:

a) $y_{n+2} - 4y_n = 9n^2$.

b) $(E^2 + 1)y_x = \sin x$. (8+12)

21. a) Prove that in the interval $-\pi < x < \pi$, $x\cos x = -\dfrac{\sin x}{2} + 2\sum_{n=2}^{\infty} \dfrac{n(-1)^n}{n^2 - 1}\sin nx$.

b) Obtain the half range cosine series for the function $f(x) = x, (0 < x < \pi)$ and

deduce that $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + ... = \dfrac{\pi^2}{8}$.

22. Diagonalize the matrix $A = \begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

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