LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – **APRIL 2013**

MT 1501 - GRAPHS, DIFF. EQU., MATRICES & FOURIER SERIES

Date: 11/05/2013   Dept. No.   Max. : 100 Marks

Time: 1:00 - 4:00

PART - A

Answer ALL the questions   **(10 X 2 = 20 Marks)**

1. Find the range and domain of $f(x) = \sqrt{x-5}$.
2. What is the axes and vertex of the parabola $y = x^2 - 2x + 3$.
3. Write the normal equations of $y = ax + b$.
4. Reduce $y = ax^n$ to linear law.
5. Define linear difference equation with an example.
6. Solve the difference equation $y_{x+3} - 3y_{x+2} - 10y_{x+1} + 24y_x = 0$.
7. Define symmetric and skew symmetric matrices.
8. Find the eigen values of the matrix $\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$.
9. Define periodic function with an example.
10. Show that $\int_c^{c+2\pi} \sin nx \, dx = 0$, when $n \neq 0$.

PART - B

Answer any FIVE questions   **(5 X 8 = 40 Marks)**

11. The demand and supply conditions under perfect competition are 16-x^2 and 2x^2+4
    respectively. Draw the graph of the function and find the equilibrium price.
12. The cost function to produce x kilograms of an item is given by
    $C(x) = 200x - 5x^2 + 0.05x^3$. Find the output at which the marginal cost is equal to the
    average cost.
13. By the method of least squares , fit a straight line to the following data

X	1	2	3	4	5
Y	14	27	40	55	68

14. Solve $y_{n+2} - 4y_{n+1} + 3y_n = 3^n + 1$

15 Form the difference equation corresponding to the family of curves $y = ax^2 + bx - 3$

16 Determine the fourier expansion of   f(x) =x, $-\pi < x < \pi$.

17 Calculate A^4 when $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$.

18 Verify Cayley Hamilton Theorem for $A = \begin{pmatrix} 3 & -1 \\ -1 & 5 \end{pmatrix}$.

PART - C

Answer any TWO questions   **(2 X 20 = 40 Marks)**

19. a) Given the cost function of an item as $C(x) = 300x - 10x^2 + \frac{x^3}{3}$. Find the output at which

   (i) Marginal Cost is minimum (ii) Average cost is minimum.

  b) Two quantities x and y are measured and corresponding values are given in the following data

X	10	20	30	40	50	60
Y	4.5	7.1	10.5	15.5	20.5	27.1

   Fit a curve of the form $a + bx + cx^2$ to the data.   (8+12)

20. a) Solve $y_{x+2} - 4y_x = 9x^2$.

  b) Solve $y_{x+2} - 8y_{x+1} + 16y_x = 4^x$.   (10+10)

21. A function $f(x)$ is defined within the range $(0, 2\pi)$ by

$$f(x) = \begin{cases} x & in\ (0, \pi) \\ 2\pi - x & in\ (\pi, 2\pi) \end{cases}$$

Express $f(x)$ as a fourier series in the range $(0, 2\pi)$ and deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

22. Diagonalize the matrix $A = \begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

$$\$\$\$\$\$\$\$$$