LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2016**

MT 1817 - ORDINARY DIFFERENTIAL EQUATIONS

Date: 02-05-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all questions. Each question carries 20 marks.

1. (a) State and prove Abel's formula. **(5)**

(OR)

(b) If Wronskian of two functions $x_1(t)$ and $x_2(t)$ on I is non-zero for at least one point on the interval I, prove that $x_1(t)$ and $x_2(t)$ are linearly independent on I. **(5)**

(c) Use the method of variation of parameters to solve $x'''(t) - x'(t) = t$. **(15)**

(OR)

(d) Check whether the given sets of functions are linearly independent.

(i) e^x, e^{-x} (ii) $1 + x,\ x^2 + x, 2x^2 - x - 3$

(iii) $\sin x, \sin 2x, \sin 3x$ on $I = [0,2\pi]$ (iv) $1,\ x,\ x^2, \ldots, x^n$ **(15)**

2. (a) Find the explicit expression for first two orders of Legendre's polynomial. **(5)**

(OR)

(b) State and prove Rodrigure's formula. **(5)**

(c) Solve by Frobenius method, $x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - y = 0$. **(15)**

(OR)

(d) Derive the generating function for Legendre's polynomial. **(15)**

3. (a) Prove that $J_n'(x) = \frac{n}{x}J_n(x) - J_{n+1}(x)$. **(5)**

(OR)

(b) Show that $\exp(\frac{1}{2}x(t - \frac{1}{t}) = \sum_{n=-\infty}^{\infty} t^n J_n(x)$. **(5)**

(c) Show that the two independent solution of the Bessel's equation may be taken to be $J_n(x)$ and $Y_n(x) = \frac{\cos n\pi\, J_n(x) - J_{-n}(x)}{\sin n\pi}$ for all values of n. **(15)**

(OR)

(d) Solve: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0, n \geq 0$. **(15)**

4. (a) Prove that all the eigen values of Strum-Liouville problem are real. **(5)**

(OR)

(b) Define Lipschitz condition and check whether the following function satisfies the same: $f(t,x) = x^{1/2}$ defined in the region $R = \{(t,x): |t| \leq 2, |x| \leq 2\}$. **(5)**

(c) State and prove Picard's theorem for initial value problem. **(15)**

(OR)

(d) Show that $x(t)$ is a solution of the equation $L[x(t)] + f(t) = 0$, $a \leq t \leq b$ if and only if $x(t) = \int_a^b G(t,s) f(s)\, ds$. **(15)**

5. (a) Explain stable solution with an example. **(5)**

(OR)

(b) State the stability behaviours of the autonomous system. **(5)**

(c) State and prove the two fundamental theorems on the stability of non-autonomous system. **(15)**

(OR)

(d) Explain the stability of the system $x' = A\,x$ by Lyapunov's method. **(15)**

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