LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2016**

MT 1818 - DIFFERENTIAL GEOMETRY

Date: 05-05-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Answer ALL the Questions:

1. a) Find the equation of the osculating plane at any point on the circular helix. **(5)**

OR

b) For the curve $\vec{x} = (u, u^2, u^3)$, find the curvature and torsion. **(5)**

c) State and prove Serret-Frenet formula and express them in terms of Darboux vector. **(15)**

OR

d) (i) Show that the necessary and sufficient condition for a curve to be plane curve is $[\bar{x}', \bar{x}'', \bar{x}'''] = 0$.

(ii) Prove that the equation of the osculating plane at the point $P(x, y, z)$ on the curve of intersection of the cylinders $x^2 + y^2 = a^2$ and $y^2 + z^2 = b^2$ is $\frac{Xx^3 - Zz^3 - a^4}{a^2} = \frac{Yy - Zz^3 - b^4}{b^2}$. **(7 + 8)**

2. a) For a curve of constant slope, prove that the necessary and sufficient condition is that the ratio of curvature to torsion is constant. **(5)**

OR

b) Prove that if the circle $lx + my + nz = 0$, $x^2 + y^2 + z^2 = 2cz$ has three point contact with the paraboloid $ax^2 + by^2 = 2z$ then $c = \frac{l^2 + m^2}{bl^2 + am^2}$. **(5)**

c) Derive the equation of evolute and involute of a curve. **(15)**

OR

d) Find the equations of the curve whose curvature and torsion are constants. **(15)**

3. a) Derive rectifying developable associated with a space curve. **(5)**

OR

b) Prove that the first fundamental form of a surface is a positive definite. **(5)**

c) Prove that the necessary and sufficient condition for the surface may be developable is that its Gaussian curvature vanish. **(15)**

OR

d) Define envelope of a curve. Find the equation of the envelope of the family of planes $3a^2x - 3ay + z = a^3$ and show that its edge of regression is the curve of intersection of the surfaces $xz = y^2$, $xy = z$. **(15)**

4. a) State and prove Meusnier theorem. **(5)**

OR

b) Find the Gaussian curvature at the point (u, v) of the anchor ring $x = (b + a\cos u)\cos v$, $y = (b + a\cos u)\sin v$, $z = a\sin u$. **(5)**

c) (i) Prove that the curves of the family $\frac{v^3}{u^2} = a\ constant$ are geodesic on a surface with metric $v^2du^2 - 2uvdudv + 2u^2dv^2$ $(u > 0, v > 0)$.

(ii) Prove that the curves $u + v =$ constant are geodesic on a surface with metric $(1 + u^2)du^2 - 2uvdudv + (1 + u^2)dv^2$. **(8 + 7)**

OR

d) Define Dupin Indicatrix. Derive the equation of Dupin Indicatrix. **(15)**

5. a) Derive Weingarten's equation. **(5)**

OR

b) Prove that the Gaussian curvature of a surface is a bending invariant. **(5)**

c) Derive the partial differential equation of surface theory. **(15)**

OR

d) State and prove the fundamental theorem of surface theory and illustrate with an example. **(15)**

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