



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2016

MT 2502 – ALGEBRA AND CALCULUS - II

Date: 21-04-2016

Dept. No.

Max. : 100 Marks

Time: 01:00-04:00

PART – A

(10 x 2 = 20 marks)

Answer ALL questions

1. If f is an odd function, show that $\int_{-a}^a f(x) dx = 0$.
2. Give the formula for finding the area of the region R in terms of polar coordinates.
3. Integrate: $\int_0^1 \int_0^2 (x^2 + y^2) dy dx$.
4. If $x = r \cos \theta$ and $y = r \sin \theta$, find the jacobian of x and y with respect to r and θ
5. Prove that $\frac{\beta(m+1, n)}{\beta(m, n)} = \frac{m}{n+1}$.
6. Evaluate $\int_0^1 x^7 (1-x)^3 dx$ using Beta and Gamma function.
7. Find the n^{th} term of the series $\frac{2}{3} + \frac{2.3}{3.5} + \frac{2.3.4}{3.5.7} + \dots \infty$.
8. State Cauchy's root test for convergence.
9. Expand $(1+x)^{-p/q}$
10. Write the expansion of $\log\left(\frac{1+x}{1-x}\right)$.

PART – B

(5 x 8 = 40 marks)

Answer any FIVE questions

11. Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$.
12. Find the length of one loop of the curve $3ay^2 = x(x-a)^2$
13. Find the area between the parabola $y = 4x - x^2$ and the line $y = x$.
14. Prove that $\overline{(n+1)} = n \overline{(n)}$. Also find $\overline{(6)}$.
15. Express $\int_0^{\pi/2} \cos^m \theta \sin^n \theta d\theta$ in terms of Beta and Gamma functions and hence evaluate $\int_0^{\pi/2} \cos^3 \theta \sin^5 \theta d\theta$.
16. Test the convergence of $\sum_1^{\infty} \sqrt{\frac{n}{n+1}} x^n$.
17. If a, b, c are three consecutive integers, show that $\log b = \frac{1}{2} \log a + \frac{1}{2} \log c + \left(\left(\frac{1}{2ac+1} \right) + \frac{1}{3} \left(\frac{1}{(2ac+1)^3} \right) + \dots \right)$.
18. Find the sum to infinity of the series $1 + \frac{1+2}{\underline{2}} + \frac{1+2+2^2}{\underline{3}} + \dots \infty$.

PART-C

(2 x 20 = 40 marks)

Answer any TWO questions

19. a) Prove that $\int_0^{\pi/4} \log(1 + \tan \theta) = \frac{\pi}{8} \log 2$.

b) Find the reduction formula for $I_n = \int \sin^n x dx$ and hence find $\int_0^{\pi/2} \sin^6 x dx$ and $\int_0^{\pi/2} \sin^5 x dx$.
(10 + 10)

20. a) Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xy dy dx$ and evaluate it.

b) Evaluate $\iint r\sqrt{a^2 - r^2} dr d\theta$ over the upper half of the circle $r = a \cos \theta$.
(12 + 8)

21. a) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

b) Test the convergence of $\sum \frac{1.3.5...(2n-1)}{2.4.6....2n} \cdot \frac{1}{n}$.
(12 + 8)

22. a) Find the sum to infinity of the series $\frac{2.4}{3.6} + \frac{2.4.6}{3.6.9} + \frac{2.4.6.8}{3.6.9.12} + \dots \infty$

b) Find the sum to infinity of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} \cdot \frac{1}{2^n}$.
(10 + 10)

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