

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2016

MT 2503 – ANALY. GEOM. OF 3D, FOURIER SERIES & NUM. THEORY

Date: 23-04-2016

Dept. No.

Max. : 100 Marks

Time: 01:00-04:00

PART – A

ANSWER ALL THE QUESTIONS

(10 x 2 = 20 marks)

1. Find the equation to the plane through (3, 4, 5) parallel to the plane $2x+3y-z+2 = 0$.
2. Write the equation of the symmetric form of the straight line.
3. Find the equation to the sphere whose centre is (2, -3, 4) and radius is 5 units.
4. Write the equation of the tangent plane to the sphere.
5. Write the formula for Fourier series.
6. Define an even function and give an example.
7. Find the remainder when 2^{1000} is divisible by 17.
8. State Fermat's theorem.
9. If a, b, c are positive, and not all equal, then show that $(a+b+c)(bc+ca+ab) > 9abc$.
10. Show that $n^n > 1 \cdot 3 \cdot 5 \dots (2n-1)$.

PART – B

ANSWER ANY FIVE QUESTIONS

(5 x 8 = 40 marks)

11. Show that, if a plane has intercepts a, b, c on the coordinate axes and is at a distance p from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$.
12. Find the image of the point (1,2,-3) on the plane $3x - 3y + 10z = 26$.
13. Find the condition that the plane $lx + my + nz = p$ may be a tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.
14. Find the equation of the sphere through the points (2,3,1), (5, -1, 2), (4, 3, -1) and (2, 5, 3).
15. Find a sine series for $f(x) = C$ in the range 0 to π .
16. Find the highest power of 3 dividing 1000!.
17. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.
18. Show that $(x^m + y^m)^n < (x^n + y^n)^m$ if $m > n$.

PART – C

ANSWER ANY TWO QUESTIONS

(2x 20 = 40 marks)

19. (a) Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$, $2x + 3y + 4z - 7 = 0$ and perpendicular to the plane $x - 5y + 3z = 5$.

(b) Find the shortest distance between the lines.

$$\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1} \quad \text{and} \quad \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}. \quad (10+10)$$

20. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, C. Find the equation of the circumcircle of the triangle ABC and determine also the coordinates of the centre and radius.

21. (a) Show that $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$ in the interval $(-\pi \leq x \leq \pi)$.

(b) Show that 8th power of any number is of the form $17m$ or $17m \pm 1$. (10+ 10)

22. (a) State and prove Wilson's theorem.

(b) If $s = a_1 + a_2 + \dots + a_n$ then show that $\frac{s}{s-a_1} + \frac{s}{s-a_2} + \dots + \frac{s}{s-a_n} > \frac{n^2}{n-1}$ unless

$$a_1 = a_2 = \dots = a_n. \quad (10+10)$$

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