LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2016**

MT 3503 – VECTOR ANALYSIS & ORDINARY DIFF. EQUATIONS

Date: 28-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART – A

ANSWER ALL THE QUESTIONS **(10 x 2 = 20 marks)**

1. Find $\nabla\phi$ at (x,y,z) if $\phi = x + xy^2 + yz^3$.
2. Find ϕ if $\nabla\phi$ is $\left(6xy + z^3\right)\vec{i} + \left(3x^2 - z\right)\vec{j} + \left(3xz^2 - y\right)\vec{k}$.
3. Show that the vector $\vec{A} = x^2z^2\vec{i} + xyz^2\vec{j} - xz^3\vec{k}$ is solenoidal.
4. Show that the vector field $\vec{f} = \left(y + y^2 + z^2\right)\vec{i} + \left(x + z + 2xy\right)\vec{j} + \left(y + 2zx\right)\vec{k}$ is conservative.
5. Find the unit vector normal to the surface $x^2 + 2y^2 + z^2 = 7$ at (1,-1,2).
6. State Green's theorem.
7. Solve $\frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0$.
8. Solve $y = \left(x - a\right)p - p^2$.
9. Solve $\left(D^2 + 5D + 6\right)y = 0$.
10. Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

PART – B

ANSWER ANY FIVE QUESTIONS **(5 x 8 = 40 marks)**

11. Find the directional derivative of $\phi = x + xy^2 + yz^3$ at (0,1,1) in the direction of the vector $2\vec{i} + 2\vec{j} - \vec{k}$.
12. Show that $\nabla^2 r^n = n\left(n+1\right)r^{n-2}$.
13. Evaluate $\iint_S \vec{A}.\ n\ dS$ if $\vec{A} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and S is the surface 2x +3y + 6z = 12 in the first octant.
14. By using Stokes' theorem evaluate the integral $I = \iint_C \left[\left(1+y\right)z\vec{i} + \left(1+z\right)x\vec{j} + \left(1+x\right)y\vec{k}\right].d\vec{r}$, where C is the circle $x^2 + y^2 = 1, z = 1$.
15. Solve $\frac{dy}{dx} - y\tan x = \frac{\sin x\cos^2 x}{y^2}$.
16. Solve $xp^2 - 2yp + x = 0$.
17. Solve $\left(D^2 - 3D + 2\right)y = Sin3x$.
18. Solve $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{\left(1-x\right)^2}$.

PART – C

ANSWER ANY TWO QUESTIONS **(2x 20 = 40 marks)**

19. (a) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$, show that $\nabla.(r^n\vec{r}) = (n+3)r^n$.

(b) In the vector field $\vec{F} = z(x\vec{i} + y\vec{j} + z\vec{k})$, evaluate $\int_C \vec{F}.d\vec{r}$ along the following curves:

(i) Curve $x = t, y = t^2, z = t^3$ from (0,0,0) to (1,1,1);

(ii) Rectilinear curve obtained by joining O(0,0,0) , A(1,0,0), B(1,1,0), C(1,1,1) by straight lines;

(iii) Straight line joining (0,0,0), (1,1,1). (8+12)

20. Verify the divergence theorem for $\vec{A} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the cylindrical region bounded by the surfaces $x^2 + y^2 = 4, z = 0, z = 3$.

21. (a) Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$.

(b) Solve $y = xp + x(1+p^2)^{1/2}$ (10+10)

22. (a) Solve $(D^2 + 4D + 5)y = e^x + x^3 + \cos 2x$.

(b) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \frac{\log x Sin(\log x) + 1}{x}$. (10+10)

$$$$$$$