LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2016**

MT 4502/MT 4500 – MODERN ALGEBRA

Date: 20-04-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART-A

ANSWER ALL THE QUESTIONS: (10x2=20marks)

1. Define partially ordered set.
2. Prove that $(ab)^2 = a^2b^2$ for all a, b in a group G if and only if G is abelian.
3. Define cyclic group and give an example.
4. Show that every subgroup of an abelian group is normal.
5. Define Homomorphism and give an example.
6. Express $(1,3,5)\ (5,4,3,2)\ (5,6,7,8)$ as a product of disjoint cycles.
7. Define Ring.
8. If F is a field show that its only ideals are {0} and F itself.
9. Prove that every field is a principal ideal domain.
10. Find all the units in Z[i].

PART-B

ANSWER ANY FIVE QUESTIONS: (5x8=40marks)

11. Show that the union of two subgroups of G is a subgroup of G if and only if one is contained in the other.
12. State and prove Lagrange's theorem.
13. If G is a group and N is a normal subgroup of G then prove that G/N is also a group under the product of subsets of G.
14. State and prove Cayley's theorem.
15. Prove that every finite integral domain is a field.
16. If R is a commutative ring with unit element whose only ideals are (0) and R itself then prove that R is a field.
17. Show that every Euclidean ring is a principal ideal domain.
18. Find a greatest common divisor of $a = 14 + 3i$ and $b = 4 + 7i$, and represent it in the form $\lambda a + \mu b$ in $Z(i)$

PART-C

ANSWER ANY TWO QUESTIONS: **(2 x 20=40marks)**

19. a) Show that a nonempty subset H of group G is a subgroup of G if and only if $a,b \in H$ implies that $ab^{-1} \in H$.

    b) If H and K are finite subgroups of a group G then prove that $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$.

20. a) Prove that every subgroup of a cyclic group is cyclic.

    b) State and prove fundamental homomorphism theorem of groups.

21. a) Let H and N be subgroups of a group G and suppose that N is normal in G . Then prove that $HN/N \simeq H/H \cap N$.

    b) Let f be a homomorphism of a ring R onto R' with kernel K. Then prove that $R/K \simeq R'$.

22. a) If R is a commutative ring with unity and P an ideal of R. Then prove that P is a prime ideal of R if and only if R/P is an integral domain.

    b) Show that an ideal of the Euclidean ring R is a maximal ideal if and only if it is generated by a prime element of R.

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