LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2016**

MT 4802 - GRAPH THEORY

Date: 18-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer all the questions. Each question carries 25 marks.

1.(a) Obtain a characterization for bipartite graphs. **(8)**

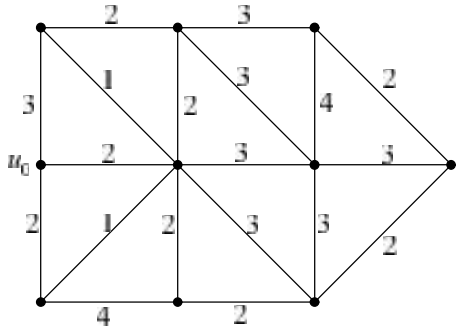
OR

(b) (i) Prove that an edge of a graph G is a cut edge if and only if it is contained in no cycle of G.
(ii) Prove that in a tree the number of edges is one less than the number of vertices. **(4 + 4)**

(c) (i) Let $d = (d_1, d_2 \ldots d_n)$ be a nonincreasing sequence of nonnegative integers, and denote the sequence $(d_2 - 1, d_3 - 1 \ldots d_{d_1+1} - 1, d_{d_1+2} \ldots d_n)$ by d'. Show that d is graphic if and only if d' is graphic.
(ii) Prove that $\tau(K_n) = n^{n-2}$. **(8 + 9)**

OR

(d) Determine the shortest paths between u_0 and all other vertices of the following graph.



(17)

2. (a) Prove that a graph G with $\nu \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally-disjoint paths. **(8)**

OR

(b) Obtain Chavatal's sufficient condition for Hamiltonian graphs. **(8)**

(c) (i) Show that every k-regular graph on $(2k + 1)$ vertices is Hamiltonian.
(ii) Prove that a nonempty connected graph is eulerian if and only if it has no vertices of odd degree. **(8 + 9)**

OR

(d) Describe the Chinese Postman problem. State Fleury's algorithm for Eulerian graphs. **(17)**

3. (a) (i) Prove with usual notation, that $\alpha' + \beta' = \nu$. **(8)**

OR

(b) Define a maximum matching and minimum covering in a graph G. Also show that the number of edges in a maximum matching is equal to the number vertices in a minimum covering in a bipartite graph. **(8)**

(c) Describe the optimal assignment problem and state Kuhn – Munkres algorithm. Obtain an optimal matching in the weighted complete bipartite graph given by the following matrix.

$$\begin{pmatrix} 5 & 5 & 4 & 1 \\ 2 & 0 & 2 & 2 \\ 4 & 4 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 1 & 3 & 3 \end{pmatrix}$$

(17)

OR

(d) State and prove the necessary and sufficient condition for a graph to have a perfect matching. **(17)**

4. (a) (i) Prove that $\delta \geq k - 1$ for a k-critical graph G.
(ii) Show that in a critical graph, no vertex cut is a clique. **(4 + 4)**

OR

(b) State and prove five color theorem. **(8)**

(c) State and prove Kuratowski's theorem. **(17)**

OR

(d) (i) State and prove Brook's theorem
(ii) Prove that if G is a tree, then $\pi_k(G) = k(k-1)^{\nu-1}$. **(10 + 7)**

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