LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2016**

MT 5407 - FORMAL LANGUAGES AND AUTOMATA

Date: 25-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

SECTION A

Answer ALL the questions: **(10x2 = 20)**

1. Define a phrase structure Grammar.
2. What is a regular set?
3. Write a grammar for the language $L(G) = \{a^n b^m / n, m \geq 1\}$.
4. Define homomorphism and ε- free homomorphism of a language.
5. If G = ({S, A}, {*a*, *b*, *c*}, S → *a*A*b*, A → *a*A*b*, A → *c*, S), find $L(G)$.
6. Define an ambiguous grammar.
7. Let $G = (N, T, P, S)$ where $N = \{S, A\}$, $T = \{a, b\}$ and P consists of the rules $\{S \to aAb, S \to a, S \to abSb, A \to bS, A \to aAAb\}$. Draw the derivation tree for the word $abab \in L(G)$.
8. Define the intersection of two languages.
9. Let $L_1 = \{x, xy, z\}$ and $L_2 = \{y, yx\}$ be the finite languages, then find (i) L_1L_2 (ii) L_2L_1.
10. Draw the state diagram for the non-deterministic finite state automaton $M = (Q, I, \delta, q_0, F)$ where $Q = \{q_0, q_1\}, I = \{0,1\}, F = \{q_1\}$ and δ is defined as follows:

	0	1
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		{

SECTION B

Answer any FIVE questions: **(5x8 = 40)**

11. Construct a context free grammar for the language $L = \{a^{2n}bc\}$. Also show that the grammar constructed generates L.
12. Let $L(G) = \{a^n b^n c^n / n \geq 1\}$. Show that $L(G)$ is accepted by the context sensitive grammar $G = (N, T, P, S)$ where $N = \{S, B\}$, $T = \{a, b, c\}$, P consists of the following productions: $S \to aSB$, $S \to abc$, $bB \to bbc$, $cB \to Bc$.
13. Write a note on Chomskian hierarchy.
14. Let G be a grammar with $S \to aSSa \mid b$. For the strings *aabbaba* and *ababbaa* find
    (i) a left most derivation and (ii) a right most derivation
15. Define Kleene closure of a language. Prove that the families of Phrase structure language, Context sensitive language, Context free language and Regular language are closed under star.
16. Let $L = \{a^n b^n / n \geq 1\}$. Give an ambiguous and unambiguous grammar to generate L.
17. Let $L = \{a^n b^m / n \neq m\}$ and $G = (N, T, P, S)$ where $N = \{S, A, B\}$, $T = \{a, b\}$ and $P = \{S \to aSb, S \to aA, A \to aA, A \to a, S \to a, S \to bB, B \to bB, B \to b, S \to b\}$ generates L. Write this grammar in Chomsky normal form.
18. Construct a finite automaton that accepts exactly those input strings of 0's and 1's that end in 11.

SECTION C

Answer any TWO questions: **(2x20 = 40)**

19. (a) If $G = (N, T, P, S)$ where $N = \{S, A, B\}$, $T = \{a, b\}$, and P consists of the following rules: $S \rightarrow aB$, $S \rightarrow bA$, $A \rightarrow a$, $A \rightarrow aS$, $A \rightarrow bAA$, $B \rightarrow b$, $B \rightarrow bS$, $B \rightarrow aBB$.

Then prove the following:

$S \Rightarrow w$ iff w consists of an equal number of a's and b's

$A \Rightarrow w$ iff w has one more a than it has b's.

$B \Rightarrow w$ iff w has one more b than if has a's

(b) Find a regular grammar to generate $L = (a, b)^*$ **(15+5)**

20. State and prove $u - v$ theorem and illustrate it with an example.

21. (a) State and prove Chomsky normal form.

(b) Let $L = \{wcw^R / w \in (a,b)^*\}$ and $G = (N, T, P, S)$ where $N = \{S\}$, $T = \{a, b, c\}$ and $P = \{S \rightarrow aSa, S \rightarrow bSb, S \rightarrow c\}$ generates L. Write this grammar in Chomsky normal form.

(10+10)

22. (a) Define a deterministic finite automaton.

(b) Construct a DFA accepting all strings over {0,1} having even number of 0's and 1's.

(c) Draw the state diagram for the following non-deterministic finite state automaton,

$M = (K, I, \delta, q_0, F)$ where $K = \{q_0, q_1, q_2, q_3\}$, $I = \{0, 1\}$, $F = \{q_3\}$, δ is defined as follows:

	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$
q_1	$\{q_3\}$	$\{q_0, q_2\}$
q_2	Φ	$\{q_3\}$
q_3	$\{q_3\}$	$\{q_3\}$

Check whether the string 11010011 is accepted by the non-deterministic finite automaton.

(3+7+10)

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