LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2016**

MT 5501 – REAL ANALYSIS

Date: 26-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART – A

ANSWER **ALL** THE QUESTIONS (10 x 2 = 20)

1. Define order complete.
2. Write the triangle inequality.
3. Define discrete metric space.
4. Define interior point.
5. Give an example of a continuous function which is not uniformly continuous.
6. Define Cauchy sequence.
7. Show that a function differential at c is also continuous at c.
8. Define local maximum.
9. Define limit superior of a real sequence.
10. State linearity property of Riemann – Stieltjes integral·

PART – B

ANSWER ANY FIVE QUESTIONS (5 x 8 = 40)

11. State and prove Minkowski's inequality.
12. If n is any positive integer, then prove that $\mathbf{N}^n$ is countably infinite.
13. Let (X, d) be a metric space. Then prove that
    (i) the union of an arbitrary collection of open sets in X is open in X.
    (ii) the intersection of an arbitrary collection of closed sets in X is closed in X.
14. Prove that the continuous image of a compact metric space is compact.
15. Let $f:(X,d_1)\to(Y,d_2)$ be uniformly continuous on X. If $\{x_n\}$ is a Cauchy sequence in X. Prove that $\{f(x_n)\}$ is a Cauchy sequence in Y.
16. State and prove Rolle's theorem .
17. Let $\{a_n\}$ be a real sequence. Then prove that
    (i) $\{a_n\}$ converges to l if and only if lim inf a_n = lim sup $a_n = l$.
    (ii) $\{a_n\}$ diverges to ∞ if and only if lim inf $a_n = +\infty$
18. State and prove the formula for Integration by parts.

PART – C

ANSWER ANY TWO QUESTIONS (2x 20 = 40)

19. (a) Prove that the set **R** is uncountable.

(b) State and prove Cauchy- Schwarz Inequality.

20. (a) Prove that the Euclidean space $\Re^k$ is complete.

(b) Every compact subset of a metric space is complete.

21. State and prove Bolzano Weierstrass theorem and deduce Intermediate value theorem.

22. (a) State and prove Taylor's theorem.

(b) Suppose (i) $f \in R(\alpha)$ on [a,b], (ii) α is differentiable on [a,b] and (iii) α^1 is continuous on [a, b]. Prove that the Riemann integral $\int_a^b f\alpha' dx$ exists and $\int_a^b f d\alpha = \int_a^b f\alpha' dx$.

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