LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **CHEMISTRY**

FIRST SEMESTER – **APRIL 2016**

CH 1808 / 5508 - QUANTUM CHEMISTRY & GROUP THEORY

Date: 06-05-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Part A

Answer ALL questions: **(10 x 2 =20)**

1. If V is a vector over a field F, Show that $(-a)v = a(-v) = -(av)$, for $a \in F$, $v \in V$.
2. Give an example of a linearly dependent set of vectors in R^2 over R.
3. Define homomorphism of a vector space into itself.
4. Define rank and nullity of a vector space homomorphism $T: U \rightarrow V$.
5. If V is an inner product space show that $< v, au + bw > = a < v, u > + \bar{b} < v, w >$ where a and b are scalars.
6. Define an orthonormal set.
7. Prove that $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
8. Define nilpotent and idempotent matrices.
9. Define Hermitian and skew-Hermitian matrices.
10. Find the rank of the matrix $A = \begin{pmatrix} 1 & 5 & -7 \\ 2 & 3 & 1 \end{pmatrix}$ over field of rational numbers.

Part B

Answer any FIVE questions: **(5 x 8 = 40)**

11. Let V be a vector space of dimension n and $u_1, u_2, u_3, ... u_r$ be linearly independent vectors in V. Show that there exists n-r new vectors $v_{r+1}, v_{r+2}, ...v_n$ in V such that $\{v_1, v_2, ..., v_n\}$ is a basis of V.
12. If V is a vector space of finite dimension that is the direct sum of its subspace U and W show that $\dim V = \dim U + \dim W$.
13. State and prove triangular inequality.
14. If V is a vector space of finite dimension and W is a subspace of V, then prove that $\dim V/W = \dim V - \dim W$.
15. Show the $T \epsilon A(V)$is invertible if and only if the constant term of the minimal polynomial for T is not zero.
16. If $T \in A(V)$ and $\lambda \in F$, then prove that λ is an eigen value of T it and only if $[\lambda I - T]$ is singular.
17. Check the consistency of the following set of equations:

    $x_1+2x_2+x_3+11$, $4x_1+6x_2+5x_3=8$, $2x_1+2x_2+3x_3=19$.
18. If $< T(v), T(v) > = < v, v >$ for all $v \epsilon V$, show that T is unitary.

Part C

Answer any TWO questions: **(2 x 20 = 40)**

19. a) Prove that the vector space V over F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$.

    b) Let V be a vector space of finite dimension and let W_1 and W_2 be subspaces of V such that $V = W_1+W_2$ and $dim\ V = dim\ W_1+dim\ W_2$. Then prove that $V = W_1 \oplus W_2$.

    (10+10)

20. If U and V are vector spaces of dimension m and n respectively over F, then prove that the vector space $Hom\ (U, V)$ is of dimension mn.

    (20)

21. Apply Gram Schmidt orthonormalization process to obtain an orthonormal basis for the subspace of R^4 generated by the vectors. (1, 1, 0, 1), (1, -2, 0, 0), (1, 0, -1, -2). **(20)**

22. a) Find the rank of $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 3 & 5 \end{bmatrix}$.

    b) If $T \in A(V)$ show that $T^* \in A(V)$ and show that

    (i) $(S+T)^* = S^* + T^*$

    (ii) $(ST)^* = T^* S^*$

    (iii) $(\lambda T)^* = \lambda T^*$

    (iv) $(T^*)^* = T$.

    (8+12)

****

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.A. DEGREE EXAMINATION – **ECONOMICS**

SECOND SEMESTER – **APRIL 2016**

LH 2084 / 5508 - ADVANCED HINDI - II

Date: 15-04-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Part A

Answer ALL questions: **(10 x 2 =20)**

23. If V is a vector over a field F, Show that $(-a)v = a(-v) = -(av)$, for $a \in F$, $v \in V$.
24. Give an example of a linearly dependent set of vectors in R^2 over R.
25. Define homomorphism of a vector space into itself.
26. Define rank and nullity of a vector space homomorphism $T: U \to V$.
27. If V is an inner product space show that $< v, au + bw >= a < v, u > + b < v, w >$ where a and b are scalars.
28. Define an orthonormal set.
29. Prove that $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
30. Define nilpotent and idempotent matrices.
31. Define Hermitian and skew-Hermitian matrices.
32. Find the rank of the matrix $A = \begin{pmatrix} 1 & 5 & -7 \\ 2 & 3 & 1 \end{pmatrix}$ over field of rational numbers.

Part B

Answer any FIVE questions: **(5 x 8 = 40)**

33. Let V be a vector space of dimension n and $u_1, u_2, u_3, ..u_r$ be linearly independent vectors in V. Show that there exists n-r new vectors $v_{r+1}, v_{r+2}, ...v_n$ in V such that $\{v_1, v_2, ..., v_n\}$ is a basis of V.
34. If V is a vector space of finite dimension that is the direct sum of its subspace U and W show that $\dim V = \dim U + \dim W$.
35. State and prove triangular inequality.
36. If V is a vector space of finite dimension and W is a subspace of V, then prove that $dim\ V/W = dim\ V - dim\ W$.
37. Show the $T \epsilon A(V)$is invertible if and only if the constant term of the minimal polynomial for T is not zero.
38. If $T \in A(V)$ and $\lambda \in F$, then prove that λ is an eigen value of T it and only if $[\lambda I - T]$ is singular.
39. Check the consistency of the following set of equations:

    $x_1+2x_2+x_3+11$, $4x_1+6x_2+5x_3=8$, $2x_1+2x_2+3x_3=19$.
40. If $< T(v), T(v) >=< v, v >$ for all $v \epsilon V$, show that T is unitary.

Part C

Answer any TWO questions: **(2 x 20 = 40)**

41. a) Prove that the vector space V over F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$.

b) Let V be a vector space of finite dimension and let W_1 and W_2 be subspaces of V such that $V = W_1+W_2$ and $dim\ V = dim\ W_1+dim\ W_2$. Then prove that $V = W_1 \oplus W_2$.

(10+10)

42. If U and V are vector spaces of dimension m and n respectively over F, then prove that the vector space $Hom\ (U, V)$ is of dimension mn.

(20)

43. Apply Gram Schmidt orthonormalization process to obtain an orthonormal basis for the subspace of R^4 generated by the vectors. (1, 1, 0, 1), (1, -2, 0, 0), (1, 0, -1, -2). **(20)**

44. a) Find the rank of $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 3 & 5 \end{bmatrix}$.

b) If $T \epsilon A(V)$ show that $T^* \epsilon A(V)$ and show that

(v) $(S+T)^* = S^* + T^*$

(vi) $(ST)^* = T^* S^*$

(vii) $(\lambda T)^* = \lambda T^*$

(viii) $(T^*)^* = T$.

(8+12)

****