LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2016**

MT 5509 – ALGEBRAIC STRUCTURE - II

Date: 29-04-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

ANSWER ALL QUESTIONS **(10 x 2 = 20)**

1. If *V* is a vector space over a field *F*, show that $(-a)v = a(-v) = -(av)$ for $a \in F, v \in V$.
2. Given that *V* is a vector space over a field *F*, prove that if v not equal to 0 then $av = 0$ implies $a=0$.
3. Define a basis of a vector space.
4. Define homomorphism of a vector space into itself.
5. Define inner product space.
6. Define orthonormal set.
7. Define skew symmetric matrix and give an example.
8. If *A and B* are Hermitian, show that *AB-BA* is skew Hermitian.
9. Define similar matrices.
10. Define adjoint of a linear transformation.

PART – B

ANSWER ANY FIVE QUESTIONS. **(5 x 8 = 40)**

11. Let *F* be an arbitrary field and *n* a positive integer. Show that the set of all ordered *n*- tuples over *F* is a vector space.
12. Prove that the union of two subspaces of a vector space *V* over *F* is a subspace of *V* if and only if one is contained in the other.
13. If *V* is a vector space of dimension *n* and *U* is a subspace of *V*, then *U* has finite dimension and *dim U* is less than or equal to *n*. Moreover, $U = V$ if and only if $dim\ U = n$.
14. A linear transformation over a vector space *V* is singular if and only if there exists a non zero element *v* in *V* such that $T(v) = 0$.
15. Prove that the regular elements of *A(V)* form a group under multiplication.
16. Show that any square matrix *A* can be expressed uniquely as the sum of a symmetric and skew symmetric matrix.
17. Find the rank of the matrix $A = \begin{pmatrix} 0 & -1 & 3 & -1 & 0 & 2 \\ -1 & 1 & -2 & -2 & 1 & -3 \\ 2 & -1 & 4 & 4 & -1 & 8 \\ 1 & -2 & 5 & 1 & -1 & 5 \end{pmatrix}$ over the field of rational numbers.
18. For what values λ of is the system of equations $\begin{aligned} x_1 + x_2 + x_3 &= 1, \\ x_1 + 2x_2 + 4x_3 &= \lambda, \\ x_1 + 4x_2 + 10x_3 &= \lambda^2 \end{aligned}$ over the rational field consistent?

PART – C

ANSWER ANY TWO QUESTIONS. **(2 x 20 = 40)**

19. Prove that a vector space V over a field F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = (0)$.
20. State and prove the fundamental homomorphism theorem for vector spaces.
21. If V is a finite – dimensional inner product space and W is a subspace of V, then prove that V is the direct sum of W and its orthogonal complement.
22. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

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