LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2016**

MT 6606 – COMPLEX ANALYSIS

[FROM 12th BATCH]

Date: 15-04-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer **ALL** questions: (10 x 2 = 20 marks)

1. Show that $f(z) = z\text{Im}z$ is differentiable only at z = 0.
2. Prove that $u = 2x - x^3 + 3xy^2$ is harmonic.
3. What do you mean by conformal mapping?
4. Under the transformation w = iz + i, show that the half plane x > 0 maps onto the half plane v >1.
5. State Cauchy's inequality.
6. Evaluate $\int_C \frac{zdz}{z-2}$ where C is the circle $|z| = 1$.
7. Write Maclaurin series expansion of Cosz.
8. Find the zeros of $f(z) = \frac{z^2+1}{1-z^2}$.
9. Find the order of the pole z = 0 for the function $\frac{1-\sin z}{z^3}$.
10. Find the residue of cotz at z = 0.

PART – B

Answer any **FIVE** questions: (5 x 8 = 40 marks)

11. If $\frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x}$ then prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$.
12. Let f be analytic in a region D and $f'(z_0) = 0$ for $z_0 \in D$. Prove that f is conformal at z_0.
13. Find the bilinear transformation which maps the points $z_1 = 0,\ z_2 = -i,\ z_3 = -1$ into the points $w_1 = i,\ w_2 = 1$ and $w_3 = 0$.
14. Evaluate $\int_C |z|\bar{z}\, dz$ where C is a closed curve consisting of the upper semicircle $|z| = 1$ and the segment $-1 \le x \le 1$.
15. State and prove Maximum Modulus theorem.
16. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$.
17. State and prove Cauchy Residue theorem.
18. Evaluate $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta}$ $(-1 < a < 1)$ by contour integration.

PART – C

Answer any **TWO** questions: (2 x 20 = 40 marks)

19. a) Derive C – R equations as necessary conditions for a function w = f(z) to be analytic. (10 marks)

b) Find the analytic function $f(z) = u + iv$ *if* $u + v = \dfrac{x}{x^2 + y^2}$ and $f(1) = 1$. (10 marks)

20. a) Prove that any bilinear transformation can be expressed as a product of translation, rotation, magnification of contraction and inversion. (8 marks)

b) State and prove Cauchy's theorem for multiply connected regions. (12 marks)

21. a) State and prove Taylor's theorem. (12 marks)

b) State and prove Liouville's theorem and deduce Fundamental theorem of algebra from it. (8 marks)

22. a) State and prove weierstrass theorem on an essential singularity. (8 marks)

b) Using the method of contour integration, evaluate $\int_{-\infty}^{\infty} \dfrac{x^2 - x + 2}{x^4 + 10x^2 + 9}\, dx.$ (12 marks)

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