LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

SECONDSEMESTER – APRIL 2017

16PMT2MC04- COMPLEX ANALYSIS

Date: 26-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

Answer all Questions. All questions carry equal marks.

1. a. Evaluate$\int_{\gamma} \frac{sinz}{z^3} dz, \gamma(t) = e^{it}, 0 \leq t \leq 2\pi$.

OR

b. State and prove Fundamental theorem of Algebra. (5)

c. State and prove Goursat's theorem.

OR

d. If γ_0 and γ_1 are two closed rectiable curves in G and$\gamma_0 \sim \gamma_1$, then prove that $\int_{\gamma_0} f = \int_{\gamma_1} f$, for every function f analytic on G. (15)

2. a. If $|a| < 1, D = \{z: |z| < 1\}, \varphi_a(\mathrm{z}) = \frac{\mathrm{z}-\mathrm{a}}{1-\bar{\mathrm{a}}\mathrm{z}}$, prove the following:
   1. φ_a is one-one map of D onto D itself.
   2. inverse of φ_a is φ_{-a}.
   3. φ_a maps ∂D onto ∂D.
   4. $\varphi_a(a) = 0, \varphi_a'(0) = 1 - |a|^2, \varphi_a'(a) = (1 - |a|^2)^{-1}$.

OR

b. State and prove Schwarz lemma. (5)

c. State and prove Hadamard's three circles theorem.

OR

d. State and prove the Riemann mapping theorem. (15)

3. a. For $Rez_n > 1$, prove that the series $\sum_{n=1}^{\infty} \log(1 + z_n)$ converges absolutely if and only if the series $\sum_{n=1}^{\infty} z_n$ converges absolutely.

OR

b. State and prove Gauss's formula. (5)

c. state and prove the weierstass factorization theorem.

OR

d. Prove that for $Rez > 1, \tau(z)\gamma(z) = \int_0^{\infty} (e^t - 1)^{-1} t^{z-1} dt$. (15)

4. a. Find the order of $exp(e^z)$ and$\exp(z^n)$.

OR

b. State and prove Mittag Leffler's theorem. (5)

c. State and prove Hadamard's factorization theorem.

OR

d. If f is an entire function of finite genus μ, then prove that f is of finite order $\lambda \leq \mu + 1$. (15)

5. a. Prove that the sum of residues of an elliptic function is zero.

OR

b. Prove that any two bases of the same module connected by a unimodular transformation. (5)

c. Prove the following:

1. $\varsigma'(z) = -\wp(z)$, $\varsigma(z)$, weierstrass zeta function and. $\wp(z)$, Weierstrass $\wp$ function
2. $\varsigma(z + w_1) = \varsigma(z) + n_1$ and $\varsigma(z + w_2) = \varsigma(z) + n_2$ where n_1 and n_2 are constants.
3. $\sigma(z + w_1) = -\sigma(z)e^{n_1\left(z+\frac{w1}{2}\right)}$ and $\sigma(z + w_2) = -\sigma(z)e^{n_2\left(z+\frac{w2}{2}\right)}$ where w_1 and w_2 are periods of Weierstrass $\wp$ function $\wp(z)$ and $\sigma(z)$, sigma function.

OR

d. Define Weierstrass $\wp$ function and derive an expression of a first order differential equation for Weierstrass $\wp$ function $\wp(z)$. (15)

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