LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

SECONDSEMESTER – APRIL 2017

16UMT2AL02- MATHEMATICS FOR STATISTICS- II

Date: 27-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

PART A

Answer all the questions: (10 X 2 = 20)

1. Find the g.l.b and l.u.b of (i) $\left\{\pi+1, \pi+\frac{1}{2}, \pi+\frac{1}{3}, \ldots\right\}$ (ii) $\{\pi+1, \pi+2, \pi+3, \ldots\}$.
2. Define Monotone sequence.
3. If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then prove that $\lim_{n\to\infty} a_n = 0$.
4. Define convergence of series.
5. If $|x-2| < 1$, prove that $|x^2-4| < 5$.
6. Define jump discontinuity.
7. State Rolle's theorem.
8. Define derivative of a function f at a point c.
9. Prove that $\int_0^{\infty} \frac{1}{e^x}\, dx$ is convergent.
10. Write any two properties of Riemann Integral.

PART B

Answer any FIVE questions: (5 X 8 = 40)

11. Find $N \in I$ such that $\left|\frac{2n}{n+3} - 2\right| < \frac{1}{5}$ and find the limit of $\left\{\frac{2n}{n+3}\right\}$.
12. If $\{s_n\}$ and $\{t_n\}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then prove that $\lim_{n\to\infty}(s_n + t_n) = L + M$.
13. If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then prove that $\sum_{n=1}^{\infty} a_n$ converges.
14. If f is continuous at a and g is continuous at $f(a)$, then prove that $g \circ f$ is continuous at a
15. State and prove the law of the mean
16. If f and g both have derivatives at $c \in R$ and $g'(c) \neq 0$ then prove that $fg, \frac{f}{g}$ has a derivative at c and $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$, $\left(\frac{f}{g}\right)'(c) = \frac{f'(c)g(c) - f(c)g'(c)}{(g(c))^2}$.
17. Prove that $\int_a^{\infty} \frac{1}{x^p}\, dx$ for $x > a > 0$ converges for $p > 1$ and diverges for $p \leq 1$.
18. Let f be a bounded function on the closed bounded interval $[a, b]$ then prove that f is Riemann integrable if and only if for every $\varepsilon > 0$, there exists a subdivision P on $[a, b]$ such that $U[f; P] - L[f; P] < \varepsilon$.

PART C

Answer any TWO questions: (2 X 20 = 40)

19. (a) If $\{s_n\}$ and $\{t_n\}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then prove that $\lim_{n\to\infty}(s_n t_n) = LM$.
(b) If A, B are subsets of S, then prove that $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$.
(12+8)

20. (a) For $a, b \in R$, show that $||a| - |b|| \leq |a - b|$. Then prove that $\{|s_n|\}$ converges to $|L|$ if $\{s_n\}$ converges to L.
(b) If $\{a_n\}$ is a sequence of positive numbers such that $a_1 \geq a_2 \geq \cdots \geq a_n \geq a_{n+1} \geq \cdots$ and $\lim_{n\to\infty} a_n = 0$ then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ is convergent.
(8+12)

21. (a) Show that $\lim_{x\to 1} \sqrt{x+3} = 2$ using definition.
(b) State and prove Taylor's theorem. (8+12)

22. (a) State and prove Binomial theorem.
(b) Let f be bounded function on $[a, b]$ and if P_1 and P_2 are any two partitions of $[a, b]$ then prove that $U[f; P_1] \geq L[f; P_2]$. (10+10)

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