LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECONDSEMESTER – APRIL 2017

MT 2502- ALGEBRA AND CALCULUS - II

Date: 04-05-2017 Dept. No. Max. : 100 Marks
01:00-04:00

PART - A

Answer **ALL** questions: **(10 X 2 = 20 marks)**

1. Prove that $\int_0^{\pi/2} \sin^2 x\, dx = \int_0^{\pi/2} \cos^2 x\, dx.$
2. Evaluate $\int_0^{\pi/2} \sin^6 x \cos^5 x dx$
3. Evaluate $\int_1^2 \int_1^2 dx\, dy$
4. Find $\frac{\partial(u,v)}{\partial(x,y)}$ where $u = x^2 + y^2, v = xy$
5. Evaluate $\int_0^1 x^7 (1-x)^8\, dx$ using Beta and Gamma functions.
6. Prove that $\Gamma(n+1) = n\Gamma(n)$, if $n > 0$
7. Test the convergence of the infinite series $1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} \ldots$
8. State Cauchy's root test for convergence of a series.
9. Prove that $\log x = \frac{x-1}{x+1} + \frac{1}{2}\frac{\left(x^2-1\right)}{(x+1)^2} + \frac{1}{3}\frac{\left(x^3-1\right)}{(x+1)^3} + \ldots$
10. Find the value of $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$

PART – B

Answer any **FIVE** questions: **(5 X 8 = 40 marks)**

11. Obtain the reduction formula for $\int \sec^n x\ dx$ where n is a positive integer.
12. Find the area bounded by one arch of the cycloid $x = a(\theta - \sin\theta)$; $y = a(1 - \cos\theta)$ and its base.
13. Evaluate $\iint xy\, dx\, dy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.
14. Show that $\beta(m, n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$.
15. Evaluate $\int_0^1 x^m (\log \frac{1}{x})^n\, dx$
16. Prove that the series $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} + \ldots$ is divergent.
17. Sum the series $1 + \frac{1}{3} + \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \ldots$

18. Sum the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \ldots$

PART – C

Answer any **TWO** questions: **(2 X 20 = 40 marks)**

19. (a). Evaluate $\int_0^{\pi/4} \log(1+\tan\theta)d\theta$.

(b). Obtain the reduction formula for $\int \sin^n x \, dx$ where n is a positive integer. **(10+10)**

20. (a). Evaluate ∭xyz dxdydz taken through the positive octant of the sphere

$x^2 + y^2 + z^2 = a^2$

(b). Change the order of integration in $\int_0^a \int_{x^2/a}^{2a-x} xy\,dy\,dx$ and evaluate it. **(7+13)**

21. (a). Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$.

(b). Express $\int_0^1 x^m (1-x^n)^p dx$ in terms of Gamma functions. **(15+5)**

22.(a). Find the sum to infinity of the series $\frac{15}{16} + \frac{15.21}{16.24} + \frac{15.21.27}{16.24.32} + \ldots\infty$

(b). Show that the series $\frac{1}{1^k} + \frac{1}{2^k} + \frac{1}{3^k} + \ldots$ is convergent when $k > 1$ and divergent when $k \leq 1$.

(10+10)

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