LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – APRIL 2017

MT 2503- ANALY. GEOM. OF 3D, FOURIER SERIES & NUM. THEORY

Date: 05-05-2017   Dept. No.   Max. : 100 Marks
01:00-04:00

PART – A

Answer ALL questions: **(10 X 2 = 20)**

1. Write the intercept form and normal form of the equation of the plane.

2. What is the angle between the plane $ax + by + cz + d = 0$ and the line $\dfrac{x - x_1}{l} = \dfrac{y - y_1}{m} = \dfrac{z - z_1}{n}$

3. Find the equation of the sphere which has its centre at the point $(-1, 2, -3)$ and radius 3 units.

4. Write the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ at the point (x_1, y_1, z_1)

5. Define odd and even functions.

6. Find the Fourier coefficient a_0 for the function $f(x) = x \sin x, \ 0 < x < 2\pi$.

7. Prove that $\left(\dfrac{1}{2} \cdot \dfrac{3}{4} \cdots \dfrac{2n - 1}{2n} \right)^{1/n} < 1$.

8. State Cauchy's inequality.

9. Define Euler's function. What is $\phi(4)$?

10. State Fermat's theorem.

PART – B

Answer any FIVE questions: **(5 X 8 = 40)**

11. Find the equation of the plane passing through the points $(3, 1, 2), (3, 4, 4)$ and perpendicular to the plane $5x + y + 4z = 0$

12. Find the shortest distance between the lines
    $\dfrac{x - 3}{-1} = \dfrac{y - 4}{2} = \dfrac{z + 2}{1}, \ \dfrac{x - 1}{1} = \dfrac{y + 7}{3} = \dfrac{z + 2}{2}$.

13. Find the equation of the sphere which has its centre at the point $(6, -1, 2)$ and touches the plane $2x - y + 2z - 2 = 0$

14. Obtain the half range cosine series for the function $f(x) = x$ in the interval $(0, \pi)$.

15. Prove that if $n > 2, (n!)^2 > n^n$

16. If x and y are positive quantities whose sum is 4, show that $\left(x + \dfrac{1}{x} \right)^2 + \left(y + \dfrac{1}{y} \right)^2 \geq 12\dfrac{1}{2}$.

17. Show that the 8th power of any number is of the form $17m$ or $17m \pm 1$.

18. Find the highest power of 3 dividing 1000! .

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PART– C

Answer any TWO questions:　　　　　　　　　　　　　　　　　　　　　**(2 X 20 = 40)**

19. a) Prove that the lines $\dfrac{x+1}{-3} = \dfrac{y+10}{8} = \dfrac{z-1}{2}$, $\dfrac{x+3}{-4} = \dfrac{y+1}{7} = \dfrac{z-4}{1}$ are coplanar.
　　Find also their point of intersection and the plane through them.

　　b) Find the image of the line $\dfrac{x-1}{2} = \dfrac{y+2}{-5} = \dfrac{z-3}{2}$ in the plane
　　　$2x - 3y + 2z + 3 = 0$.　　　　　　　　　　　　　　　　　　　**(8 + 12)**

20. a) Find the equation of the sphere which touches the sphere
　　$x^2 + y^2 + z^2 - 6x + 2z + 1 = 0$ at the point $(2, -2, 1)$ and passes through the origin.
　　b) Find the equation of the sphere through the four points
　　$(2, 3, 1), (5, -1, 2), (4, 3, -1)$ and $(2, 5, 3)$.　　　　　　　　　**(10 + 10)**

21. a) Determine the Fourier expansion for $f(x) = \begin{cases} -\pi & in\ (0,\ \pi) \\ x - \pi in\ (\pi,\ 2\pi) \end{cases}$
　　and show that $\sum_{r=1}^{\infty} \dfrac{1}{(2r+1)^2} = \dfrac{\pi^2}{8}$

　　b) Prove that $8xyz < (y+z)(z+x)(x+y) < \dfrac{3}{8}(x^3 + y^3 + z^3)$　　**(12 + 8)**

22. a) Show that if x and y are both prime to the prime number n, then $x^{n-1} - y^{n-1}$ is divisible by n.

　　b) State Wilson's theorem and prove that $18! + 1$ is divisible by 437.
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