LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2017

MT 2962- ACTUARIAL MATHEMATICS

Date: 03-05-2017   Dept. No.   Max. : 100 Marks
01:00-04:00

Answer ALL Questions: **(5 *20=100)**

1. (a) Express tp_x and tq_x in terms of $S(x)$.

(OR)

(b) Define deferred probability and derive the expression for deferred probability.   **(5)**

(c) If $S(x) = 1-\frac{x^2}{100}, 0 \leq x \leq 10$, then calculate

   (i) $F_X(x)$, (ii) $_t p_4$, (iii) $_{2/2}q_4$ and (iv) probability density function of $T(4)$.

(d) Find A if $\mu_x = A + e^x$ and $_{0.50}p_0 = 0.50$.   **(10+5)**

(OR)

(e) Derive force of mortality.

(f) If $S(x) = 1 - \frac{x^2}{100}, 0 \leq x \leq 10$. find (i) find median future life time of $x = 5$, $t^q_x = 0.5$.

(g) Derive probability density function of $T(x)$.   **(4+6+5)**

2. (a) Suppose a survival model is defined by the value of p_x.

   x :   0   1   2   3   4
   p_x :   0.9   0.8   0.6   0.3   0 .

   What are the corresponding values of $S(x)$ for $x = 0, 1, 2, 3, 4$ and 5.

(OR)

(b) Derive an expression for $_nD_x$.   **(5)**

(c) An aviary of birds which has a constant intake of 100 new born birds per year experience the following mortality rates:

x	0	1	2	3	4	5
$q(x)$	0.3	0.1	0.2	0.4	0.7	1

   (i)   What is the expected total number of birds in the aviary at any time?
   (ii)   What is the expected number living between ages 1 and 4?

(d) Prove that $l_{x+t} = l_x - {}_t d_x$.   **(10+5)**

1

(e) Derive the expression for l_x ,d_x ,L_x ,T_x ,e_x and tabulate the values of l_x ,d_x ,L_x ,T_x ,e_x where $q_0 = 0.2$, $q_1 = 0.45$, $q_2 = 0.50$, $q_3 = 0.65$, $q_4 = 1$ and taking $l_0 = 100$.

(f) Given that $p_{40} = 0.999473$, calculate $_{0.4}q_{40.2}$ under the assumption of under distribution of death.

(10+5)

3. (a) In how many years will a sum of money double itself when it is compounded?

(OR)

(b) Find the principle,if the amount with compound interest of 5% per annum is 3969 for the period of 2 years. **(5)**

(c) If the probability density function of the future life time T is given by

$$g(t) = \begin{cases} \dfrac{1}{80}, & 0<t<80 \\ 0 & elsewhere \end{cases}, \text{ then calculate}$$

(i) the net single premium at a force of interest δ,

(ii) the variance and

(iii) the 90th percentile.

(d) Find the amount to which 1000 will accumulate the rate of interest corresponding to an effective rate of discount at 3% per annum for 8 years. **(10+5)**

(OR)

(e) Assume that each of 100 independent lives is of age x, is subject to a constant force of mortality $\mu = 0.04$ and is insured for a death benefit amount of 10 units, payable at the moment of death. The benefit payments are to be withdrawn from an investment fund earning interest at a rate $\delta = 0.06$. Calculate the minimum amount to be collected at t=0, so that the probability is approximately 0.95 that sufficient funds will be on hand to withdraw the benefit payment at the death of each individual.

(f) Give an account of endowment insurance policy. **(10+5)**

4. (a) Derive the certain and accumulated value in annuities.

(OR)

(b) Explain Term –annuity due? **(5)**

(c) An alumni association has 50 members, each of age x. It is assumed that all lives are independent. It is decided to contribute Rs. R to establish a fund to pay a death benefit of Rs . 10, 000/- to each member. Benefits are to be payable at the moment of death. It is given that $\overline{A}_x = 0.06$ and $^2\overline{A}_x = 0.01$. Using normal approximation, find R so that with probability 0.95 the fund will be sufficient to pay the death benefit.

(d) Rs. 3000 is deposited at a bank if January 1st of each year from $2001-2009$. What is the accumulated value of this fund on December 31, 2009 at 3% annual rate of interest?

(10+5)

(OR)

(e) A loan of Rs. 50, 000/- is taken on January 12, 2014. It has to be repaid by 15 equal installment payable yearly at the beginning of the year based on an 8% annual rate interest, determine the amount of installment.

(f) Prove that $Var(\ddot{a}_{\overline{k+1|}}) = \dfrac{{}^2A_x - \left(A_x\right)^2}{d^2}$.

(8+7)

5. (a) Calculate $\ddot{a}_x$ where it is given that ${}_{10}E_x = 0.40$, ${}_{\overline{10|}}\ddot{a}_x = 7$ and $\ddot{S}_{x:\overline{10|}} = 15$.

(OR)

(b) For a whole life insurance with unit benefit, calculate $\bar{P}(\bar{A}_x)$ and $var\,(L)$ with the assumptions that the force of mortality is constant $\mu = 0.04$ and force of interest $\delta = 0.06$. **(5)**

(c) For a three year temporary life annuity due one (30), you are given:

(i) $S(x) = 1 - \frac{x}{80}, 0 \le x < 80$

(ii) $i = 0.05$

(iii) $Y = \begin{cases} \ddot{a}_{\overline{k+1|}} & \text{if } k = 0,1,2 \\ \ddot{a}_{\overline{3|}} & \text{if } k = 3,4,5 \end{cases}$. Calculate $var\,(Y)$.

(d) Given (i) ${}_{\overline{10|}}\ddot{a}_x = 4.0$, (ii) $\ddot{a}_x = 10.0$ (iii) $\ddot{S}_{x:\overline{10|}} = 15.0$, (iv) $\vartheta = 0.94$. Calculate $A'_{x:\overline{10|}}$.

(10+5)

(OR)

(e) For a fully continuous whole life insurance 1 on (x). Calculate $\bar{P}(\bar{A}_x)$ given the following:

(i) Premiums are determine using the equivalence principle.

(ii) $\frac{var\,[Z]}{var\,[L]} = 0.36$ and

(iii) $\bar{a}_x = 10$.

(f) If ${}_{k|}q_x = c\left(0.96\right)^{k+1}$, $k = 0,1,2,...$ where c=0.04/0.96 and i=0.06, calculate P_x and Var(L).

(8+7)

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