LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTHSEMESTER – APRIL 2017

MT 4503- ALGEBRAIC STURUCTURE - I

Date: 21-04-2017 Dept. No. Max. : 100 Marks
09:00-12:00

PART – A

ANSWER ALL THE QUESTIONS **(10 x 2 = 20)**

1. Define an equivalence relation.
2. Prove that $(ab)^2 = a^2b^2$ for all a and b in a group G if and only if G is abelian.
3. Prove that every cyclic group is abelian.
4. Define a normal subgroup of a group.
5. Define (i) Homomorphism (ii) Isomorphism.
6. Define transposition.
7. Define a division ring.
8. If F is a field show that its only ideals are {0} and F itself.
9. Prove that every field is a principal ideal domain.
10. Define a maximal ideal.

PART – B

ANSWER ANY FIVE QUESTIONS **(5 x 8 = 40)**

11. If G is a group, then prove that (i) the identity element of G is unique (ii) every 'a' $\in$ G has a unique inverse in G.
12. Prove that every subgroup of a cyclic group is cyclic.
13. State and prove Lagrange's theorem.
14. Prove that a subgroup N of a group G is a normal subgroup of G if and only if the product of two left cosets of N in G is again a left coset of N in G.
15. State and prove Cayley's theorem.
16. Prove that every finite integral domain is a field.
17. Let R be a commutative ring with unity, and 'P' an ideal of R. Prove that P is a prime ideal of R if and only if R/P is an integral domain.
18. Let R be a Euclidean ring. Prove that any two elements a and b in R have a greatest common divisor d which can be expressed in the form $\lambda a + \mu b$, for some $\lambda, \mu \in \Re$.

PART – C

ANSWER ANY TWO QUESTIONS **(2x 20 = 40)**

19. (a) If H and K are subgroups of G, then prove that HK is a subgroup of G if and only if HK=KH

(b) If H and K are finite subgroups of a group G, prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$. **(8+12)**

20. (a) If G is a group and N is a normal subgroup of G, prove that G/N is also a group under the product of subsets of G.

(b) Prove that the ring of integers is a principal integral domain.

(10 +10)

21. (a) Let H and N be subgroups of a group G, and suppose that N is normal in G. Prove that $\dfrac{HN}{N} \cong \dfrac{H}{H \cap N}$.

(b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field. **(10+10)**

22. (a) Let R be a Euclidean ring. Prove that every non-zero element of R is either unit in R or can be uniquely written as a product of a finite number of prime elements of R.

(b) Prove that Z(i) is Euclidean ring. **(10 +10)**

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