LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTHSEMESTER – APRIL 2017

MT 5508 / MT 5502 - LINEAR ALGEBRA

Date: 20-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

PART A

ANSWERALL THE QUESTIONS $\mathbf{10 \times 2 = 20}$***marks***)

1. Define a vector space V over a field F.
2. Show that the vectors (0,1,1) , (0,2,1)and (1,5,3) in R^3 are linearly independent over R ,the field of real numbers.
3. Prove that the vectors (1,0,0), (1,1,0) and (1,1,1) form a basis of R^3, where R is the field of real numbers.
4. Verify that $T : R^2 \to R$ defined by $T(a,b) = ab$ for all $a,b \in R$ is a vector space homomorphism .
5. Normalize $(1+2i, 2-i, 1-i)$ in C^3 relative to the standard inner product.
6. If $T \in A(V)$ and $\lambda \in F$ and λ is an eigenvalue of T then prove that $\lambda I - T$ is singular.
7. Show that $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
8. Define trace of a matrix and give an example.
9. Find the rank of the following matrix over the field of rational numbers $\begin{pmatrix} 2 & 0 \\ 1 & 1 \\ 1 & -1 \end{pmatrix}$
10. If $T \in A(V)$ is Hermitian, then prove that all its eigenvalues are real.

PART B

ANSWERANY FIVE QUESTIONS ($\mathbf{5 \times 8 = 40}$***marks***)

11. Prove that the union of two subspaces of a vector space V over F is a subspace of V if and only if one is contained in the other.
12. If S and T are subsets of a vector space V over F, then prove the following:
    i) S is subspace of V if and only if $L(S) = S$.
    ii) $S \subseteq T$ implies that $L(S) \subseteq L(T)$.
    iii) $L(L(S)) = L(S)$.
13. Let V be a finite dimensional vector space and suppose that one basis has n elements and another basis has m elements .Then prove that $m = n$.
14. If V is a vector space of finite dimension and W is a subspace of V ,then prove that $\dim V/W = \dim V - \dim W$.

15. If $\lambda_1, \lambda_{2,} \ldots \lambda_n$ are distinct eigenvalues of $T \in A(V)$ and if $v_1, v_{2,} \ldots v_n$ are eigenvectors of T belonging to $\lambda_1, \lambda_{2,} \ldots \lambda_n$, respectively then prove that $v_1, v_{2,} \ldots v_n$ are linearly independent over F.

16. If $A, B \in F_n$ and If $\lambda \in F$, then prove that

    i) $(\lambda A)^t = \lambda A^t$

    ii) $(A^t)^t = A$

    iii) $(A + B)^t = A^t + B^t$

    iv) $(AB)^t = B^t A^t$

17. Investigate for what values of λ, μ the system of equations $x_1 + x_2 + x_3 = 6, x_1 + 2x_2 + 3x_3 = 10, x_1 + 2x_2 + \lambda x_3 = \mu$ over the rational field has a) no solution b) a unique solution c) an infinite number of solutions.

18. a) If $T \in A(V)$ is skew-Hermitian, prove that all of its eigenvalues are pure imaginaries.

    b)Prove that the eigenvalues of a unitary transformation are all of absolute value 1.

PART C

ANSWERANY TWO QUESTIONS $(2 \times 20 = 40 marks)$

19. a) Prove that the vector space V over F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = (0)$.

    b) If V and W are two n-dimensional vector spaces over F. Then prove that any isomorphism T of V onto W maps a basis of V onto a basis of W.

20. If W_1 and W_1 are subspaces of a finite dimensional vector space V, prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

21. State and prove Gram-Schmidt orthonormalization process.

22. a) Show that any square matrix A can be expressed uniquely as the sum of a symmetric matrix and a skew-symmetric matrix.

    b) Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V onto an orthonormal basis of V.

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