LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – APRIL 2017

MT 6608- DISCRETE MATHEMATICS

Date: 24-04-2017 Dept. No. Max. : 100 Marks
09:00-12:00

PART-A

Answer all the questions **(10 x 2=20)**

1. Construct the truth table for $P \wedge Q$.
2. Write the duals of (a) $(P \wedge Q) \vee T$ (b) $\neg(P \vee Q) \wedge \left(P \vee \neg\left(Q \wedge \neg S\right)\right)$.
3. Write down the min terms of P and Q.
4. Define Tautology.
5. Define semigroup.
6. Define monoid.
7. Define lattice.
8. Define lattice homomorphism.
9. Define Boolean algebra.
10. State De Morgan's law for Boolean Algebra.

PART-B

Answer any FIVE questions **(5 x 8=40)**

11. Construct the truth table for $(P \to Q) \wedge (Q \to P)$.
12. Show that $\left(\neg P \wedge \left(\neg Q \wedge R\right)\right) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.
13. Obtain the principal disjunctive normal form of $P \to \left((P \to Q) \wedge \neg\left(\neg Q \vee \neg P\right)\right)$.
14. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$.
15. Prove that for any commutative monoid $(M, *)$, the set of all idempotent elements of M forms a sub monoid.
16. Let $(L, \leq)$ be a lattice. Then prove that for any $a, b, c \in L$, the inequality $a \oplus (b * c) \leq (a \oplus b) * c$ holds.
17. State and prove the Isotonicity property in a lattice.
18. Obtain the values of the Boolean forms (a) $x_1 * x_2$ (b) $x_1 * (x_1' \oplus x_2)$ (c) $x_1 \oplus (x_1 * x_2)$

PART-C

Answer any TWO questions **(2 x 20=40)**

19. (a) Show that $\left((P \vee Q) \wedge \neg\left(\neg P \wedge \left(\neg Q \vee \neg R\right)\right)\right) \vee \left(\neg P \wedge \neg Q\right) \vee \left(\neg P \wedge \neg R\right)$ is a tautology.

(b) Obtain the principal disjunctive normal form of $\left(\neg P \rightarrow R\right) \wedge (Q \leftrightarrow P)$. **(10+10)**

20. (a) Show that the following premises are inconsistent.

I If Jack misses many classes through illness, then he fails in high school.

II If Jack fails high school, then he is uneducated.

III If Jack reads a lot of books, then he is not uneducated.

IV Jack misses many classes through illness and reads a lot of books.

(b) Prove that the composition of semigroup homomorphism is also a semigroup homomorphism.

(10+10)

21. (a) State and prove any four properties of lattice.

(b) Define sub Boolean Algebra. **(18+2)**

22. (a) Write down the following Boolean expressions in an equivalent sum of the products canonical form in three variables x_1, x_2 and x_3 (i) $x_1 * x_2$ (ii) $x_1 \oplus x_2$ (iii) $(x_1 \oplus x_2)' * x_3$.

(b) Define the following (i) Complete lattice (ii) Bounded lattice (iii) Complemented lattice (iv) Distributive lattice.

(10+10)

****