LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FOURTHSEMESTER – APRIL 2018

16PMT4MC05- CLASSICAL MECHANICS

Date: 25-04-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer **ALL** questions:

1. (a) State and prove D'Alembert's principle. **(5)**

(OR)

(b) Find the equation of motion for simple pendulum of length l and mass m by using Lagrange's equations. **(5)**

(c) State and prove Lagrange's equations in holonomic system. **(15)**

(OR)

(d) A particle of mass m moves in a conservative force field. Find the Lagrangian and the equations of motion in cylindrical coordinates (ρ, φ, z). **(15)**

2. (a) Obtain the Hamilton's equation of motion for a projectile assuming that the axes are attached to the earth. **(5)**

(OR)

(b) Derive the Hamilton's principle from the Lagrange's equation of motion. **(5)**

(c) State and prove Rouths' Procedure. **(15)**

(OR)

(d) State and prove the conservation theorem for linear momentum in Lagrangian formulation. **(15)**

3. (a) With usual notations prove the following:

(i) $[u,v]_{q,p} = -[v,u]_{q,p}$

(ii) $[u,u]_{q,p} = [v,v]_{q,p} = 0.$

(iii) $[u+v,w] = [u,w] + [v,w].$ **(5)**

(OR)

(b) Show that the transformation $P = \sqrt{2q}e^{-\alpha}\sin p$, $Q = \sqrt{2q}e^{\alpha}\cos p$ is canonical. **(5)**

(c) State and prove principle of Least action. **(15)**

(OR)

(d) State and prove Poincare theorem. **(15)**

4. (a) Find the relation between infinitesimal contact transformation and Poisson bracket. **(5)**

(OR)

(b) Derive Hamilton-Jacobi equation. **(5)**

(c) Define Poisson bracket of two dynamical variables. Show that for three such variables u, v, w the Jacobi identity: $\big[u,[v,w]\big] + \big[v,[w,u]\big] + \big[w,[u,v]\big] = 0$ is satisfied. **(15)**

(OR)

(d) State and prove Liouville's theorem in phase space. **(15)**

5. (a) If $\{u_l, u_i\}$ is a Lagrange bracket and $[u_l, u_j]$ is a Poisson bracket, then prove that $\sum_{l=1}^{2n}\{u_l, u_i\}\left[u_l, u_j\right] = \delta_{ij}$. **(5)**

(OR)

(b) Write short note on Action and Angle variables. **(5)**

(c) Solve the problem of one dimensional Harmonic oscillator, where the Hamiltonian is given by $H = \frac{p^2}{2m} + \frac{1}{2}Kq^2$. **(15)**

(OR)

(d)UsingHamilton-Jacobi's equation, solve the Kepler's problem for the Hamiltonian given by $H = \frac{1}{2m}\left[p_r^2 + \frac{p_\theta^2}{r^2}\right] - \frac{\lambda}{r}$. **(15)**

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