LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2018**

17PMT2MC01/ MT 2810 – ALGEBRA

Date: 17-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Answer ALL the questions

1. a) If $O(G) = p^2$ where p is a prime number, then show that G is abelian.

OR **(5)**

b) Prove that a group of order 72 is not simple.

c) If p is a prime number such that p^α divides order of G then prove that G has a subgroup of order p^α.

OR **(15)**

d) State and prove Cauchy's theorem and prove that the number of *p-sylow* subgroups in G is of the form $1 + kp$.

2. a) State and prove Division Algorithm.

OR **(5)**

b) If $f(x)$ and $g(x)$ are primitive polynomials then $f(x)g(x)$ is also a primitive polynomial.

c) Let $f(x) = a_0 + a_1x + \ldots + a_nx^n$ be a polynomial with integer coefficients. Suppose for some prime number p, $p \nmid a_n$, $p \mid a_1$, $p \mid a_2, \ldots, p \mid a_0$, $p^2 \nmid a_0$ then prove that $f(x)$ is irreducible over rationals.

(8)

d) If $f(x)$ and $g(x)$ are two nonzero polynomials, then prove that $deg\big(f(x)g(x)\big) = deg\big(f(x)\big) + deg(g(x))$. **(7)**

OR

e) If the primitive polynomial $f(x)$ can be factored as the product of two polynomials having rational coefficients then prove that it can be factored as the product of two polynomials with integer coefficients. **(8)**

f) If $O(G) = p^n$ where p is a prime number then prove that $Z(G) \neq (e)$. **(7)**

3. a) If *a*, *b* in *K* are algebraic over *F*, then prove that $a \pm b$, ab and *a/b (if b* $\neq 0$) are algebraic over *F*.

OR **(5)**

b) Find the degree of $\sqrt{2} + \sqrt{3}$ over Q.

c) The element $a \in K$ is said to be algebraic over *F iff F(a)* is a finite extension over *F*.

OR **(15)**

d) If L is the finite extension of K and K is the finite extension of F then prove that L is the finite extension of F. **(8)**

e) If L is the finite extension of F and K is the subfield of L which contains F then prove that $[K:F]$ divides $[L:F]$. **(7)**

4. a) Find the degree of the splitting field x^3-2 over Q and x^4+x^2+1 over Q.

OR **(5)**

b) Prove that K is a normal extension of F iff K is a splitting field of some polynomial over F.

c) State and prove fundamental theorem of Galois Theory.

OR **(15)**

d) Let K be a normal extension of F and let H be a subgroup of $G(K,F)$, $K_H=\{x \in K \,/\, \sigma(x) = x \ \forall\ \sigma \in H\}$ is a fixed field of H then prove that (i) $[K:K_H] = O(H)$, (ii) $H = G(K,K_H)$. In particular, $H = G(K,F)$, $[K:F] = O(G(K,F))$.

5. a) Let G be a finite abelian group such that $x^n = (e)$ is satisfied by atmost n elements of G for every n then prove that G is a cyclic group.

OR **(5)**

b) Prove that for every prime number p and every integer m, there exists a field having p^m elements.

c) Prove that any finite division ring is necessarily a commutative field.

OR **(15)**

d) Prove that S_n is not solvable for $n \geq 5$ and verify S_3 is solvable.

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