LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **PHYSICS**

SECONDSEMESTER – APRIL 2018

17/16UMT2AL01- MATHEMATICS FOR PHYSICS - II

Date: 28-04-2018 Time: 01:00-04:00 Dept. No.

Max.: 100 Marks

(10X2 = 20)

Part A (Answer ALL questions)

- 1. Evaluate $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$.
- 2. Find the value of $\int_{0}^{\pi/2} \cos^{6} x \, dx$

3. Prove that
$$\int_{0}^{a} f(x)dx = \int_{0}^{a} f(a-x)dx$$

- 4. Write any two properties of beta function.
- 5. Solve $x\sqrt{1+y^2} + y\sqrt{1+x^2} \frac{dy}{dx} = 0$.
- 6. Show that the following differential equation is exact.

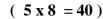
$$(x^2 - x + y^2)dx - (ye^y - 2xy)dy = 0$$

- 7. Find $\frac{\partial(u,v)}{\partial(x,y)}$ when u = x + y and v = x y.
- 8. Evaluate $\int_0^1 \int_0^2 \int_0^3 xyz \, dx \, dy \, dz$.
- 9. Prove that $\nabla \Box r = 3$ and $\nabla \times r = 0$ where *r* is the position vector of the point P(x,y,z).
- 10. State Stoke's theorem.

Part B (Answer any FIVE questions)

- 11. Evaluate $\int \frac{(3x+1)dx}{(x-1)^2(x+3)}$.
- 12. Establish the reduction formula for $I_n = \int \sin^n x dx$ (n being a positive integer) and hence find the

value of
$$\int_{0}^{\frac{\pi}{2}} \sin^{5} x \, dx$$
.
13. Evaluate
$$I = \int_{0}^{\frac{\pi}{2}} \log \sin x \, dx$$





14. Prove the following.

i. $\Gamma(n+1) = n!$, where n is a positive integer.

ii.
$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$
.

15. Solve $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\cos 2x$ 16. Solve $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$

17. By transforming into polar co-ordinates evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ (b > a)

18. Evaluate $\iint_{S} F \Box n \, dS$, where $F = zi + xj - y^2 z k$, and S is the surface of the cylinder $x^2 + y^2 = 1$ included in the first octant between the planes z=0 and z=2.

 $(2 \times 20 = 40)$

Part C (Answer any TWO questions)

19. a) Prove that
$$\int_{0}^{\frac{\pi}{4}} \log(1 + \tan \theta) d\theta = \frac{\pi}{8} \log 2$$

- b) Solve $(D^2 + 16)y = 2e^{-3x} + \cos 4x$ (10 + 10)
- 20. a) Express $I = \int_{0}^{1} x^{m} (1 x^{n})^{p} dx$ in terms of Gamma function and evaluate the integral $I = \int_{0}^{1} x^{5} (1 - x^{3})^{10} dx$
 - b). Evaluate $\int_{0}^{1} x^{m} (\log \frac{1}{x})^{n} dx (10 + 10)$

21.a) By changing the order of integration , evaluate $\int_{0}^{\infty} \int_{x}^{\infty} \frac{e^{-y}}{y} dx dy$

b) By changing into polar co-ordinates evaluate the integral $\int_{0}^{2a} \int_{0}^{\sqrt{2ax-x^{2}}} (x^{2} + y^{2}) dx dy \qquad (8+12)$

22. a) Evaluate
$$\int \frac{6x+5}{\sqrt{6+x-2x^2}} dx$$

b) Find by Green's theorem the value of $\int_C (x^2 y dx + y dy)$ along the closed curve C formed by the curves $y^2 = x$ and y=x between (0, 0) and (1, 1). (10+10)