LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION –**MATHEMATICS**

FIRST SEMESTER – **APRIL 2018**

MT 1503– ANALYTICAL GEOMETRY OF 2D,TRIG. & MATRICES

Date: 26-04-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART-A

Answer all questions: (10 x 2 =20)

1. Write the expansion of $\tan n\theta$ in powers of $\tan\theta$.
2. Solve approximately in radians $\sin\left(\frac{\pi}{3}+x\right)=0.87$.
3. Show that $\tan h^{-1}x=\frac{1}{2}\log_e\left(\frac{1+x}{1-x}\right)$.
4. Find the logarithms of $x+iy$.
5. Define similar matrices.
6. State Cayley-Hamilton theorem.
7. Find the pole of the line $5x-2y+3=0$ with respect to the parabola $y^2=2x$.
8. Show that the sum of the squares of two conjugate semi-diameters of an ellipse is constant.
9. Find the asymptotes of the hyperbola $3x^2-5xy-2y^2+17x+y+14=0$.
10. Find the distance between the points in polar coordinates.

PART-B

Answer any FIVE questions (5 x 8 = 40)

11. Prove that $\frac{\sin 9\theta}{\sin\theta}=256\,cos^8\theta-448\,cos^6\theta+240\,cos^4\theta-49\,cos^2\theta+1$.
12. Expand $sin^6\theta cos^2\theta$ in a series of cosines of multiples of θ.
13. Separate into real and imaginary parts $tan^{-1}(x+iy)$.
14. Find the general value of $\log_{(-3)}(-2)$.
15. Calculate A^4 when $A=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$.
16. A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose centre is C meets the circle $x^2+y^2=a^2+b^2$ at Q and Q'. Prove that CQ and CQ' are conjugate diameters of the ellipse.
17. Show that the locus of the intersection of tangents to $y^2=4ax$ which intercept a constant length d on the directrix is $(y^2-4ax)(x+a)^2=d^2x^2$.
18. The asymptotes of a hyperbola are parallel to $2x+3y=0$ and $3x-2y=0$.Its centre is at $(1,2)$ and it passes through the point $(5,3)$.Find its equation and its conjugate.

PART-C

Answer any **TWO** questions: (2 x 20 = 40)

19. (a). Expand $\cos 9\theta$ in powers of $\cos\theta$. (7+7+6)
(b). Find the expansion of $sin^n\theta$ when n is a positive integer.

(c) Find $\lim_{x\to\frac{\pi}{2}} \frac{\sin x+\cos 2x}{\cos^2 x}$.

20. (a) If$\cosh u = \sec\theta$,show that $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$. (10+10)

(b) If $\sin(A+iB) = x+iy$,prove that
(i) $\frac{x^2}{sin^2A} - \frac{y^2}{cos^2A} = 1.$
(ii) $\frac{x^2}{cosh^2B} + \frac{y^2}{sinh^2A} = 1.$

21. (a) Find the Eigen values of the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix}$.

(b) Verify the Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.(10+10)

22.(a) The polar of a point P with respect to the parabola $y^2 = 4ax$ meets the curve in Q and R. Show that if P lies on the line $lx+my+n=0$,then prove that the middle point of QR lies on the parabola $l(y^2-4ax)(lx+my+n)=0$.

(b) Show that $r^2 - kr\cos(\theta-\alpha) + kd = 0$ represents a system of coaxial circles for different values of k.Find the coordinates of the limiting points and the equation of the common radialaxis. (10+10)

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