LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – APRIL 2018

MT 1819- PROBABILITY THEORY & STOCHASTIC PROCESSES

Date: 02-05-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART –A

Answer ALL questions 10 * 2 = 20

1. Define distribution function of a random variable.
2. State Boole's inequality..
3. Write the sample space for tossing two fair coins simultaneously.
4. Find $E(X)$, if X is an exponential random variable.
5. Write the probability distribution function of exponential distribution.
6. Define weak law of large numbers.
7. Write any two properties of normal distribution.
8. Define consistent estimators.
9. How do you understand Markov chain.
10. State Rao-Blackwell theorem.

PART-B

Answer any FIVE questions 5 * 8 = 40

11. State and prove Baye's theorem.
12. A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour.
13. Find the moment generating function of gamma distribution and hence find its mean and variance.
14. A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find k, (ii) Evaluate $P(X < 6), P(X \geq 6)$, and $P(0 < X < 5)$ (iii) If $P(X \leq a) > \frac{1}{2}$, find the minimum value of a.

15. Let$x_1, x_2, \dots, x_n$ be a random sample from a distribution with probability density function $f(x,\theta) = e^{-(x-\theta)}, \theta < x < \infty$. Find a sufficient ststistic forθ.
16. Find the maximum likelihood estimator for the parameter λ of a poisson distribution on the basis of a sample size n.
17. State and prove Chebyshev's inequality.
18. If $x \geq 1$, is the critical region for testing $H_0: \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population $f(x,\theta) = \theta e^{-\theta x}, 0 \leq x < \infty$, obtain the values of type I and type II errors.

PART –C

Answer any TWO questions 2 * 20 = 40

19.a) Let X be a continuous random variable with p.d.f given by

$$F(x) = \begin{cases} kx, & 0 \le x < 1 \\ k, & 1 \le x < 2 \\ -kx + 3x, & 2 \le x < 3 \\ 0, & elsewhere \end{cases}$$

(i)Determine the constant *k*, (ii) Determine *F(x)*, the c.d.f and (iii) If $x_1, x_{2,} x_3$ are three independent observation from *X*, what is the probability that exactly one of these three numbers is larger than 1.5?

b) Let A and B be two events in a sample space S and are not disjoint, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

20.a) Find the marginal distribution of X and Y, and the conditional distribution of Y for X=x.

b) Obtain the first and second central moments of Beta distribution of second kind.

21.a) State and prove Cramer-Rao inequality.

b) State and prove Neyman- Pearson lemma.

22. Briefly explain a time dependent general birth and death process in stochastic process.

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