## PART - A

## Answer ALL questions

1. Find the first derivative of $x e^{x}$ with respect to $x$.
2. Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ when $\mathrm{u}(\mathrm{x}, \mathrm{y})=x^{3} y+9 x^{4}-2 y^{2}$.
3. Evaluate $\int\left(3+2 x-x^{4}\right) d x$.
4. Define consumer surplus.
5. What is a slack variable?
6. Write the dual of the following LPP.

Maximize $Z=2 x_{1}+x_{2}+5 x_{3}$
Subject to the constraints $\quad x_{1}+5 x_{2}+x_{3} \leq 12$

$$
\begin{aligned}
& 2 x_{1}-x_{2}-x_{3} \leq 3 \\
& 2 x_{1}-2 x_{2}-3 x_{3} \leq 8, x_{1}, x_{2}, x_{3} \geq 0
\end{aligned}
$$

7. What is an unbalanced transportation problem?
8. What is an assignment problem?
9. Define project in network analysis.
10. Define critical path in a network.

## $\underline{\text { PART - B }}$

Answer any FIVE questions
$(5 \times 8=40)$
11. If $y=\left(x+\sqrt{1+x^{2}}\right)^{m}$, show that $\left(1+x^{2}\right) y_{2}+x y_{1}=m^{2} \mathrm{y}$.
12. Find the maximum value of the function $\frac{\log x}{x}$ for $x>0$.
13. Evaluate $\int \frac{(6 x+5)}{\sqrt{6+x-2 x^{2}}} d x$.
14. Solve the following LPP by the graphical method.

$$
\text { Maximize } Z=3 x_{1}+4 x_{2}
$$

Subject to the constraints $x_{1}+x_{2} \leq 450$

$$
\begin{aligned}
& 2 x_{1}+x_{2} \leq 600 \\
& \text { and } x_{1}, x_{2} \geq 0 .
\end{aligned}
$$

15. Determine consumer surplus and producer surplus under pure competition for the demand function $p=36-x^{2}$ and supply function $p=6+\frac{x^{2}}{4}$, where $p$ is the price and $x$ is the quantity.
16. Consider the problem of assigning four jobs to four persons. The assignment costs are given as follows.

Persons

$$
\begin{aligned}
& \\
& \\
& \\
& \boldsymbol{J}_{\mathbf{1}} \\
& \\
& \boldsymbol{J}_{\mathbf{2}} \\
& \boldsymbol{J}_{\mathbf{3}} \\
& \boldsymbol{J}_{\mathbf{4}}
\end{aligned}\left(\begin{array}{rrrrr}
5 & 7 & 116 \\
8 & 5 & \boldsymbol{P}_{\mathbf{1}} & \boldsymbol{P}_{\mathbf{2}} \boldsymbol{P}_{\mathbf{3}} & \boldsymbol{P}_{\mathbf{4}} \\
4 & 7 & 107 \\
10 & 4 & 8 & 3
\end{array}\right)
$$

Find the optimal assignment.
17. Draw the network for the project whose activity and precedence relationships are given below.

| Activity | A | B | C | D | E | F | G | H | I |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Predecessor | - | A | A | - | D | B,C,E | F | E | G,H |

18. Find the initial transportation cost of the following matrix using North West corner method and least cost method.

|  |  | Destination |  |  |  | Availability |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ |  |
|  | $O_{1}$ | 1 | 2 | 1 | 15 | 30 |
| $\stackrel{50}{5}$ | $\mathrm{O}_{2}$ | 3 | 3 | 2 | 1 | 50 |
|  | $\mathrm{O}_{3}$ | 15 | 2 | 5 | 9 | 20 |
|  | mand | 20 | 40 | 30 | 10 |  |

## PART - C

## Answer any TWO questions

19.a) Find the maxima and minima of the function $u(x, y)=y^{4}+2\left(x^{2}-y^{2}\right)-x^{4}$.
b) Evaluate $\int_{0}^{\pi / 2} \frac{\cos ^{4} x}{\sin ^{4} x+\cos ^{4} x} d x$.
20.Solve the following linear programming problem by simplex method.

$$
\text { Maximize } Z=3 x_{1}+2 x_{2}+5 x_{3}
$$

Subject to constraints $x_{1}+4 x_{2} \leq 420$

$$
3 x_{1}+2 x_{3} \leq 460
$$

$$
x_{1}+2 x_{2}+x_{3} \leq 430
$$

$$
\text { and } \quad x_{1}, x_{2}, x_{3} \geq 0
$$

21. Construct a network for the project whose activities and the three time estimates namely optimistic time
$\mathbf{t}_{o}$, most likelihood time $\mathfrak{t}_{m}$ and pessimistic time $\mathbf{t}_{p}$ of these activities (in weeks) are given below. Compute
a) Expected duration of each activity
b) Expected variance of each activity and also fine the critical path of the project and the expected project duration.

| Activity | $\mathbf{1 - 2}$ | $\mathbf{2 - 3}$ | $\mathbf{2 - 4}$ | $\mathbf{3 - 5}$ | $\mathbf{4 - 5}$ | $\mathbf{4 - 6}$ | $\mathbf{5 - 7}$ | $\mathbf{6 - 7}$ | $\mathbf{7 - 8}$ | $\mathbf{7 - 9}$ | $\mathbf{8 - 1 0}$ | $\mathbf{9 - 1 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{t}_{\boldsymbol{w}}$ | 3 | 1 | 2 | 3 | 1 | 3 | 4 | 6 | 2 | 1 | 4 | 3 |
| $\mathbf{t}_{\boldsymbol{m}}$ | 4 | 2 | 3 | 4 | 3 | 5 | 5 | 7 | 4 | 2 | 6 | 5 |
| $\mathbf{t}_{\boldsymbol{p}}$ | 5 | 3 | 4 | 5 | 5 | 7 | 6 | 8 | 6 | 3 | 8 | 7 |

22. Find the optimal transportation cost of the following transportation problem by modified distribution (MODI) method.

|  |  | Destination |  |  |  | Availability |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ |  |
|  | $O_{1}$ | 11 | 20 | 7 | 8 | 50 |
| 䫆 | $\mathrm{O}_{2}$ | 21 | 16 | 20 | 12 | 40 |
|  | $\mathrm{O}_{3}$ | 8 | 12 | 18 | 9 | 70 |
|  | and | 30 | 25 | 35 | 40 |  |

