LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2018**

MT 4503– ALGEBRAIC STURUCTURE - I

Date: 09-05-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART –A

ANSWER ALL THE QUESTIONS: **(10 X 2 = 20)**

1. Define equivalence class.
2. Let $G = \{1, -1\}$ under the multiplication of real numbers, then prove that G is an abelian group of order 2.
3. If G is a finite group of order n and $a \in G$, then prove that a^n= e .
4. State Fermat theorem.
5. Define an automorphism of a group.
6. Prove that (1 , 2 , 3 , 4 , 5) (1 , 2 , 3) is an even permutation.
7. If a ring R has a unit element, then prove that it is unique.
8. Define left-ideal.
9. Define Euclidean ring.
10. Define prime element.

PART - B

ANSWER ANY FIVE QUESTIONS: **(5 X 8 = 40)**

11. If H and K are any two nonempty subsets of a group G, then prove that $(HK)^{-1} = K^{-1}H^{-1}$.
12. If H and K are subgroups of G, then prove that $H \cap K$ is a subgroup of G.
13. If G is a group and $a, x \in G$, then prove that a and $x^{-1}ax$ have the same order.
14. If f is a homomorphism of a group G into a group G', then prove that kernel of f is a normal subgroup of G.

15.Let H and N be subgroups of a group G, and suppose that N is a normal in G, then prove that $\frac{HN}{N} \simeq \frac{H}{H\cap N}$.

16.If R is a ring , for all $a, b \in R$, then prove that

(i) $a0 = 0a = 0.$

(ii) $(-a)b = a(-b) = -(ab).$

(iii) $(-a)(-b) = ab.$

17.Prove that every Euclidean ring is a principal ideal domain (PID).

18.Let R be a commutative ring with unity, and P an ideal of R .Then prove that P is a prime ideal of R if and only if $\frac{R}{P}$ is an integral domain.

PART – C

ANSWER ANY TWO QUESTIONS: (2 X 20 = 40)

19.a) If H and K are a finite subgroups of a group G, then prove that

$$o(HK) = \frac{o(H)o(K)}{o(H \cap K)}.$$

b) Prove that every subgroup of a cyclic group is cyclic.

20. a) If G being a group, let $Z(G) = \{ z \in G / zx = xz \ for\ all\ x \in G\}$,show that$Z(G)$is a normal subgroup of G.

b) Prove that any infinite cyclic group G is isomorphic to the group Z of integers under addition.

21. a) Let f be a homomorphism of a group G onto a group G' with kernel K. Let N' be a normal subgroup of G' and let $N = \{x \in G / f(x) \in n'\}$. Prove that $G/N \cong G'/N'$.

b) Prove that every finite integral domain is a field.

22.a) State and prove fundamental theorem of ring homomorphism.

b) An ideal of the Euclidean ring R is a maximal ideal of R if and only if it is generated by a prime element of R.

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