# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034 

Date: 04-05-2018
B.Sc.DEGREE EXAMINATION -MATHEMATICS

FIFTH SEMESTER - APRIL 2018
MT 5510- STATICS

Time: 09:00-12:00
Dept. No. $\square$ Max. : 100 Marks

## SECTION - A

## Answer ALL questions

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(10 \times 2=20)
$$

1. When do you say that a concurrent system of forces is in equilibrium?
2. What is meant by composition of forces and resolution of a force?
3. Define like parallel forces and unlike parallel forces.
4. Define moment of a force.
5. State the formula for coordinates of the centre of gravity a rigid body.
6. What is the centre of gravity of a thin uniform rod?
7. State the principle of virtual work for a system of coplanar forces acting on a rigid body.
8. When a body is said to be in neutral equilibrium?
9. Define catenary.
10. What is a suspension bridge?

## SECTION - B

Answer any FIVE questions.
11. The magnitude of the resultant of two given forces of magnitudes $P$ and $Q$ is $R$. The magnitude of the resultant is doubled either when the force of magnitude $Q$ is doubled or reversed in the direction. Prove that $P: Q: R=\sqrt{2}: \sqrt{3}: \sqrt{2}$.
12. Two strings $A B$ and $A C$ are knotted at $A$, where a weight $W$ is attached. If the weight hangs freely and in the position of equilibrium, with $B C$ horizontal, $A B: B C: C A=2: 4: 3$, show that the tensions in the strings are $\frac{7 W}{2 \sqrt{15}}$ and $\frac{11 W}{4 \sqrt{15}}$.
13. Find the resultant of two unlike parallel forces with unequal magnitudes.
14. Find the centre of gravity of a uniform solid tetrahedron.
15. Find the centre of gravity of a uniform solid right circular cone.
16. A string of length a forms the shorter diagonal of a rhombus of four uniform rods, each of length $b$ and weight W which are hinged together. If one of the rods be supported in a horizontal position, prove that the tension in the string is $\frac{2 W\left(2 b^{2}-a^{2}\right)}{b \sqrt{\left.4 b^{2}-a^{2}\right)}}$.
17. State and prove Lami's Theorem.
18. A string of length $2 l$ hangs over two small smooth pegs in the same horizontal level. Show that , if $h$ is the sag in the middle, the length of either part of the string that hangs vertically is $\mathrm{h}+\mathrm{I}-2 \sqrt{h l}$.

## SECTION - C

Answer any TWO questions
$(2 \times 20=40)$

19 (a) A weight is supported on a smooth plane inclined at the angle $\alpha$ with the horizon, by a string inclined to the vertical at the angle $\beta$. If the inclination of the plane is increased to $\gamma$ and the inclination of the string with the vertical is unaltered, the tension in the string is doubled in supporting the weight. Prove that $\quad \cot \alpha-2 \cot \gamma=\cot \beta$.
(b)Two beads of weight W and $\mathrm{W}^{\prime}\left(\mathrm{W}^{\prime}>\mathrm{W}\right)$ can slide on a smooth circular wire in a vertical plane. They are connected by a light string which subtends an angle $2 \beta$ at the centre of the circle when the beads are in equilibrium on the upper half of the wire. Prove that the inclination $\alpha$ of the string to the horizontal is given by

$$
\tan \alpha=\frac{W^{\prime}-W}{W^{\prime}+W} \tan \beta .
$$

20 (a) State and prove Varignon's theorem on moments.
(b) A uniform ladder of length 1 and weight W rests with its foot on the rough ground and its upper end against a smooth wall, the inclination to the vertical being $\alpha$. A force P is applied horizontally to the ladder at a point distance c from the foot so as to make the foot approach the wall. Prove that P must exceed $\quad \frac{l W}{l-c}\left(\mu+\frac{1}{2} \tan \alpha\right)$ where $\mu$ is the coefficient of friction at the foot.

21 (a) Find the centre of gravity of the area bounded by $y$ - axis, the line $y=2 a$ and the cycloid $x=a(\theta+$ $\sin \theta), \mathrm{y}=\mathrm{a}(1-\cos \theta)$ that lies in the first quadrant.
(b) A rod lies in equilibrium with its ends on two smooth planes inclined at angles $\alpha, \beta$ to the horizontal , the planes intersecting in a horizontal line. Show that the inclination of the rod to the horizontal is tan ${ }_{-1}^{-1} \frac{\sin (\alpha \sim \beta)}{2 \sin \alpha \sin \beta}$.

22(a) A string of length $\iota$ hangs between two points not in the same vertical line and the tangents at the end points are inclined at an angle $\alpha$ and $\beta$ with the horizontal. Show that the height of one extremity above the other is $\frac{l \sin \frac{\alpha+\beta}{2}}{\cos \frac{\alpha-\beta}{2}}$.
(b)A uniform chain of length $2 \iota$ has its ends attached to two points in the same horizontal line at a distance 2 a apart. If $\iota$ is only a little greater than a, show that the tension in the chain is approximately equal to a weight of the chain of length $\sqrt{\frac{a^{3}}{6(l-a)}}$ and the sag or depression of the lowest point of the chain below its end is $\frac{1}{2} \sqrt{6 a(l-a)}$ nearly.

