LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2019**

16/17/18UMT2MC01– ALGEBRA AND CALCULUS - II

Date: 02-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Part A

Answer ALL the Questions: **(10x2 = 20)**

1. Evaluate $\int \left(ax + \frac{b}{x^2}\right) dx$.
2. State any two properties of definite integrals.
3. Evaluate $\int_0^3 \int_0^2 (x^2 + y^2)\, dy\, dx$.
4. When Cartesian coordinates are transformed into polar coordinates, what is the value of theJacobian?
5. Define Gamma functions.
6. Evaluate $\int_0^{\frac{\pi}{2}} sin^7\theta\, d\theta$.
7. State D'Alembert's Ratio test.
8. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(n+1)^2}{n!}$.
9. Write the expansion of $(1 – x)^{-2}$.
10. Show that $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\cdots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\cdots} = \frac{e-1}{e+1}$.

Part B

Answer Any FIVE Questions: **(5x8 = 40)**

11. Sum to infinity the series $\sum_0^{\infty} \frac{5n+1}{(2n+1)!}$.
12. Prove that $log\sqrt{12} = \left(\frac{1}{2}+\frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4}+\frac{1}{5}\right)\frac{1}{4^2} + \cdots$.
13. Test the convergence of the series $\sum_1^{\infty} \left(\frac{n}{n+1}\right)^{\frac{1}{2}} x^n$.
14. Find the area of the circle $r = 2a\cos\theta$.
15. Find the length of the arc of the parabola $r(1+ \cos\theta) = 2a$ cut off by the latus rectum.
16. Prove that $\beta(m, n) = \beta(n, m)$.
17. Prove that $\int_0^{\infty} e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$.
18. Evaluate $\iint xydxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$ by transforming into polar coordinates.

Part C

Answer any TWO Questions: **(2 x 20 = 40)**

19\. a. Show that the series $\sum \frac{\{(n+1)r\}^n}{n^{n+1}}$ is convergent if $r < 1$ and divergent if $r \geq 1$.

b. Show that $\frac{1}{1.2.3} + \frac{1}{3.4.5} + \frac{1}{5.6.7} + \cdots \infty = \log 2 - \frac{1}{2}$. **(10+10)**

20\. a. Derive the relation between Beta and Gamma functions.

b. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **(15+ 5)**

21\. a. Change the order of integration and evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dy\, dx.$

b. Find the area of the surface of the solid generated by rotating the cardioid $r = a(1 + cos\theta)$ about its line of symmetry. **(10+10)**

22\. a. Derive the reduction formula for $\int sin^n x dx$.

b. Show that $\int_0^{\frac{\pi}{4}} \log(1 + \tan^{-}) \, d\theta = \frac{\pi}{8} \log 2.$

(10+ 10)

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