LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2019**

16/17/18UMT2MC02– ANA. GEO. OF 3D, FOURIER SERIES AND NUMBER THEORY

Date: 04-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer ALL questions. **(10 ò 2 = 20)**

1. Find the equation of the plane through (3,4,5) and parallel to the plane 2x+ 3z – z =0.
2. Find the equation of the straight line passing through the points origin and(5, -2, 3).
3. Find the equation of the sphere with centre (-1, 2, -3) and radius 3 units.
4. Find the general equation of a sphere passing through the circle

$x^2+y^2+z^2$ +2ux+2vy+2wz+d =0 ; ax+by+cz+ k =0.

5. Give an example of a function which is neither odd nor even.
6. Find a_0 of the Fourier series for e^x in the interval - π< x< -π.
7. Find the number of integers less than n and prime to it when n=729.
8. If a≡b (mod m) and c≡d(mod m), show that a+c ≡ b+d (mod m).
9. If a , b , c are positive and not all equal, show that (a+b+c) (bc+ca+ab) > 9abc.
10. State Wierstrass inequality.

PART – B

Answer any FIVE questions **(5 ò 8 = 40)**

11. Find the equation of the plane through the points (3 ,1, 2), (3, 4, 4) and perpendicular to the plane 5x + y +4z = 0.
12. Find the symmetric form of the equation of the line of intersection of the planes 3x-2y+z=1 and 5x+4y - 6z = 2.
13. Find the equation of the sphere having the circle x^2 +y^2 +z^2-2x +4y-6z +7 =0 ; 2x-y+2z =5 as great circle.
14. Find the equation of the sphere through the points(2,3,1) , (5,-1,2) , (4,3,-1) and (2,5,3).

15. Express f(x) $=\frac{1}{2}(\pi - x)$ as a Fourier series to be valid in the interval 0 to 2π.

16. Obtain cosine series for $f(x) = \begin{cases} \cos x, & o < x < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < x < \pi \end{cases}$.

17. State and Prove Fermat's theorem.

18. If $a_1, a_2, \ldots a_n$ is an arithmetic progression, show that $a_1^2 a_2^2 \ldots a_n^2 > a_1^n a_n^n$.

PART – C

Answer any TWO questions. **(2 ò 20 = 40)**

19.(a) Show that the origin lies in the acute angle between the planes x+2y+2z = 9, 4x – 3y + 12z +13=0. Find the planes bisecting the angles between them and point out which bisects the obtuse angle.

(b)Find the equation of the image of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane 3x-3y+10z - 26=0.

20. (a)Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$ and $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar. Find the point of intersection and the plane through them.

(b) Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x - 4y = 0$; x+2y+3z =8 and touches the plane 4x+3y =25.

21. Obtain the Fourier series for the function f(x) = x^2 and deduce that

(i) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12}$ (ii) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$.

22. (a) State and prove Wilson's theorem.

(b) Show that if $s = a_1 + a_2 + a_3 + \ldots + a_n$, show that $\frac{s}{s-a_1} + \frac{s}{s-a_2} + \cdots + \frac{s}{s-a_n} > \frac{n^2}{n-1}$.

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