LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2019**

16/17/18UMT1MC02– ANALYTICAL GEOMETRY OF 2D, TRIG. MATRICES

Date: 05-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

ANSWER ALL QUESTIONS $(10X2 = 20)$

1. Expand $\tan n\theta$ in powers of $\tan\theta$.
2. Write down the expansion of $\sin\theta$ and $\cos\theta$ in ascending powers of θ.
3. Prove that $\cosh^2 x - \sinh^2 x = 1$.
4. Separate $\tanh(1+i)$ into real and imaginary parts.
5. State Cayley Hamilton theorem.
6. Find the eigenvalues of the matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.
7. Define Conic.
8. Find the centre of the ellipse $9x^2 + 25y^2 - 18x + 100y - 116 = 0$.
9. What is the length of the latus rectum of the hyperbola?
10. Write down the equation for distance between points (r_1, θ) and (r_2, θ).

PART – B

ANSWER ANY FIVE QUESTIONS $(5X8 = 40)$

11. Expand $\cos^6\theta$ and $\sin^5\theta$ in series of cosines of multiples of θ.
12. Show that $\sin\left(\frac{\pi}{3}+x\right) = 0.87$
13. Evaluate $\lim_{x\to\frac{\pi}{2}} \frac{x+\cos 2x}{\cos^2 x}$.
14. If $\tan(x+iy) = u+iv$, prove that $\frac{u}{v} = \frac{\sin 2x}{\sinh 2y}$.
15. Verify Cayley Hamilton theorem for $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$.

16. Find the eigenvalues and eigenvectors of $A=\begin{pmatrix}10 & -2 & -5\\ -2 & 2 & 3\\ -5 & 3 & 5\end{pmatrix}$.

17. Find the focus, vertex and the directrix of the parabola $y^2-4x-6y+5=0$.

18. Obtain the combined equation of the pair tangents from the point (x_1,y_1) to the parabola $y^2=4ax$.

PART – C

ANSWER ANY TWO QUESTIONS $(2X20=40)$

19. (a) Express $\cos 3\theta$ interms of $\sin\theta$

(b) Expand $\sin^3\theta\cos^5\theta$ in a series of sines of multiples of θ.

20. (a) If $\cos(x+iy)=\cos\theta+i\sin\theta$, prove that $\cos 2x+\cosh 2y=2$ and $\sin^3\theta\cos^5\theta$ in a series of sines of multiples of θ.

(b) Find the real part and imaginary part of $\tan^{-1}(x+iy)$.

21. Diagonalise the matrix $A=\begin{pmatrix}2 & 0 & 4\\ 0 & 6 & 0\\ 4 & 0 & 2\end{pmatrix}$

22. (a) If the focus, centre and eccentricity of an ellipse are respectively $(2,3)$, $(3,4)$ and $\frac{1}{2}$, find its equation. **(10+10)**

(b) Show that the feet of the perpendiculars from the origin on the sides of the triangle formed by the points $\theta=\alpha,\theta=\beta$ and $\theta=\gamma$ on the circle $r=2a\cos\theta$ lie on the straight line.