LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2019**

MT 3503– VECTOR ANALYSIS & ORDINARY DIFF. EQUATIONS

Date: 25-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Part – A (10 x 2 =20 marks)

Answer all questions :

1) Find the directional derivative of $\emptyset = x + xy^2 + yz^3$ at (0, 1, 1) in the direction of the vector $2\vec{i} + 2\vec{j} - \vec{k}$.
2) Show that the vector $A = x^2z^2\vec{i} + xy\,z^2\vec{j} - xz^3\vec{k}$ is solenoidal.
3) What is the condition for a vector field to be conservative?
4) Prove that ∇ x ∇ ∅ = 0.
5) State Stoke's theorem.
6) Using Gauss divergence theorem show that $\iint_s r.n\,ds = 4\pi a^2$ if S is the surface of the sphere x^2 $y^2 + z^2 = a^2$.
7) Solve $y = (x – a) p – p^2$.
8) Solve $p^2 – 5p + 6 = 0$.
9) Solve $(D^2 + 5D + 6) y = 0$.
10) Find the particular integral of $(D^2 – 3D + 2) y = e^{5x}$.

Part – B (5x8=40 Marks)

Answer any five questions :

11) Show that $\nabla^2 r^n = n\,(n + 1)r^{n-1}$.
12) Find the curl and divergence of the vector $\vec{F} = xy^2\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$ at (1, – 1, 1).
13) Evaluate $\iiint_v \nabla . F\,dv$ *if* $\vec{f} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and if V is the volume of the region enclosed by the cube $0 \leq x, y, z \leq 1$.
14) If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20\,xz^2\vec{k}$ and C is the curve $x = t$, $y = t^2$, $z = t^3$ from (0, 0, 0) to (1, 1, 1).
15) Solve $xp^2 – 2yp + x = 0$.
16) Solve $\frac{dy}{dx} - y\tan x = \frac{\sin x\,cos^2 x}{y^2}$
17) Solve $(D^2 + D + 1) y = e^{– 2x}$.
18) Solve $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$ using method of variation of parameters.

Part – C (2 x 20 = 40 Marks)

Answer any two questions :

19) a) If $\emptyset = x^2y^3z^4$ Find (i) $\nabla . \nabla\emptyset$ (ii) $\nabla \times \nabla\emptyset$.

b) Find the equation of the tangent plane to the surface $x^2 + 2y^2 + 3z^2 = 6$ at the point (1 -1, 1).

(10 + 10)

20) Verify divergence theorem for $A = (x + y)\vec{i} + x\vec{j} + z\vec{k}$ taken over the region V of the cube bounded by the planes x =0, x=1, y=0, y=1, z=0, z=1.

21) a) Solve $[D^2 + 2D + 5]\, y = xe^x$

b) Solve $(D^2 + 4D + 5)\, y = e^x + \cos 2x$. **(10 + 10)**

22) Solve $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + \log x.$

****