LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION –**MATHEMATICS**

FIFTH SEMESTER – **APRIL 2019**

MT 5505– REAL ANALYSIS

Date: 15-04-2019 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A ($10 \times 2 = 20$)

ANSWER **ALL** THE QUESTIONS:

1. Show that the sets $\mathbb{Z}$ and $\mathbb{N}$ are similar.
2. Differentiate countable and uncountable sets.
3. Define a) Cauchy sequence and b) Complete metric space.
4. Define compact sets.
5. Define adherent point.
6. Give an example of a continuous function which is not uniformly continuous.
7. Define complete metric space and give an example of a space which is not complete.
8. If a real-valued function f has a derivative at $c \in \mathbb{R}$, prove that f is continuous at c.
9. Let $f: [0,1] \to \mathbb{R}$ be such that

$$f(x) = \begin{cases} 0, & if\ x\ is\ rational \\ 1, & if\ x\ is\ irrational \end{cases}$$

   Check whether the function is Riemann integrable or not.
10. Give an example of a function which is not Riemann Stieltjesintegrable.

PART – B ($5 \times 8 = 40$)

ANSWER ANY **FIVE** QUESTIONS:

11. a) Write the field axioms of the set of real numbers.

    b) Find the limit supremum and limit infimum of $(-1)^n\left(1+\frac{1}{n}\right)$.
12. State and prove Minkowski's inequality.
13. Let Y be a subspace of a metric space (X, d). Show that a subset A of Y is open in Y if and only if $A = Y \cap G$ for some open set G in X.
14. Prove that a closed subset of a complete metric space is also complete.
15. State and prove Bolzano's theorem.
16. State and prove Lagrange's mean value theorem.
17. Suppose $f \in \mathbb{R}(\alpha)$ on $[a, b]$. Show that $\alpha \in \mathbb{R}(f)$ on $[a, b]$ and

$$\int_a^b f\,d\alpha + \int_a^b \alpha\,df = f(b)\alpha(b) - f(a)\alpha(a).$$

18. Let $f: [a, b] \to \mathbb{R}$ be such that f is differentiable on (a, b) with $|f'(x)| \le K$ for all $x \in (a, b)$ for some positive constant K and f is continuous at the end points a and b. Then prove that f is of bounded variation on $[a, b]$.

PART – C $(2 \times 20 = 40)$

ANSWER ANY **TWO** QUESTIONS:

19. a)i) If $\mathcal{F}$ is a countable collection of pairwise disjoint countable sets, then prove that $\bigcup_{F\in\mathcal{F}} F$ is countable.

ii) Given a countable family $\mathcal{F}$ of sets, then prove that one can find a countable family $\mathcal{G}$ of pairwise disjoint sets such that $\bigcup_{F\in\mathcal{F}} F = \bigcup_{G\in\mathcal{G}} G$.

b) Prove that every integer $n > 1$ can be expressed as a product of primes in unique way but for the order of the factors.

20. a) Let S be a compact subset of a metric space X. Then prove that

i) S is bounded and closed.

ii) every infinite subset of S has an accumulation point in S.

b) State and prove Heine-Borel theorem.

21. a) Show that continuous image of a compact metric space is compact.

b) Show that continuous image of a connected metric space is connected.

22. a) State and prove Rolle's theorem.

b) Let f and g be functions of bounded variation defined on $[a, b]$. Then prove that the functions $f + g$, $f - g$ and fg are also bounded variation on $[a, b]$. Also prove further that $V_{f\pm g} \leq V_f + V_g$ and $V_{fg} \leq AV_f + BV_g$, where $A = \underset{a \leq x \leq b}{Sup} |g(x)|$ and $B = \underset{a \leq x \leq b}{Sup} |f(x)|$.

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