LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2019**

MT 4503– ALGEBRAIC STURUCTURE - I

Date: 11-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART –A

Answer ALL questions **(10 X 2 = 20 Marks)**

1. Define equivalence relation.
2. Prove that $(ab)^2 = a^2b^2$ for all $a, b \in G$ where G is an abelian group.
3. If G is a finite group of order *n* and $a \in G$, Prove that $a^n = e$.
4. Show that every subgroup of an abelian group is normal
5. Define automorphism of a group
6. Let *Z* be the set of all integer and $h: Z \to Z$ may be defined by *h(x)* = *2x*. Show that it is a group homomorphism.
7. State any two properties of rings.
8. If f is a homomorphism of a ring R into a ring R', Prove that $f(-a) = -f(a)$ for all $a \in R$.
9. Prove that every field is a principle ideal domain.
10. What is a Gaussian integer?

PART – B

Answer any FIVE questions. **(5 X 8 = 40 Marks)**

11. If G is a group, Prove that
    (i) the identity element of G is unique
    (ii) every $a \in G$ has a unique inverse in G.
12. Show that the set Q^+ of all positive rational numbers forms a group under the operation $*$ defined by $a * b = \frac{ab}{2}$ for all $a, b \in Q^+$.
13. Prove that every subgroup of a cyclic group is cyclic.
14. Prove that a subgroup *N* of a group *G*is a normal subgroup of *G* if and only if the product of two left cosets of *N* in *G* is again a left coset of *N* in *G*.
15. Prove that every group is isomorphic to a group of permutations.
16. If R is a commutative ring with unit element whose only ideals are (0) and R itself, Prove that R is a field.
17. Let *R* be a commutative ring with unity, and *M* an ideal of *R*. Prove that if *M* is a maximal ideal of *R* then*R*/*M* is a field.
18. Prove that every Euclidean ring is a principal ideal domain.

PART – C

Answer any TWO questions. **(2 X 20 = 40 Marks)**

19. (i) If H and K are subgroups of G, Prove that HK is a subgroup of G if and only if $HK = KH$. **(8)**

(ii) If H and K are finite subgroups of a group G, Prove that $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$. **(12)**

20. (i) State and prove Lagrange's theorem. **(12)**

(ii) Show that the union of two subgroups of a group G is a subgroup of G if and only if one is contained in other. **(8)**

21. (i) State and prove fundamental theorem of homomorphism of a group. **(12)**

(ii) Prove that $A(G)$, the set of automorphisms of a group G is also a group. **(8)**

22. (i) Prove that every finite integral domain is a field. **(10)**

(ii)Prove that $Z(i)$ is a Euclidean ring. **(10)**

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