LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2022**

16/17UMT6MC04 – GRAPH THEORY

Date: 21-06-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

Part A (Answer ALL questions) **(10 x 2 = 20)**

1. State Königsberg bridge problem.
2. Define complete graph.
3. Give one example for null graph.
4. Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.
5. What is a Hamiltonian circuit?
6. Define a tree.
7. A graph with atleast one vertex is also called a tree. True or False. Justify A
8. Define non-separable graph with example.
9. What is embedding in graph?
10. Explain chromatic number.

Part B (Answer any FIVE questions) **(5 x 8 = 40)**

11. If a graph G (connected or disconnected) has exactly two vertices of odd degree, prove that there must be a path joining these two vertices.
12. Explain the following terms with examples: (i) Walk, (ii) Open andClosed walk, (iii) path, (iv) length and cycle
13. Define the following operations on graphs with two examples:
    a. Ring sum
    b. Complement
    c. Decomposition **(4+2+2)**
14. If n is an odd number and $n \geq 3$, prove that in a complete graph with n vertices there are $(n-1)/2$ edge-disjoint Hamiltonian circuits.
15. Prove that any connected graph with n vertices and $n-1$ edges is a tree.
16. Show that the vertex connectivity of a graph cannot exceed the edge connectivity of G.
17. In any simple, connected planar graph with f regions, n vertices and e edges $(e > 2)$, show that the following inequalities must hold: (i) $e \geq \frac{3}{2}f$; (ii) $e \leq 3n-6$. **(4+4)**
18. List the properties of chromatic number.

Part C (Answer any TWO questions) **(2 x 20 = 40)**

19. (a) Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.
    (b) Prove that a connected graph G is an Euler graph iff all vertices of G are of even degree. **(10+10)**
20. (a) Show that the number of vertices of odd degree in a graph G is always even with n vertices and e edges.
    (b) Show that a tree with n vertices has $n-1$ edges. **(10+10)**
21. (a) Show that the distance between any two vertices of a connected graph is a metric.
    (b) A connected graph with n vertices and e edges has $n-1$ branches, then show that G has $e-(n-1)$ chords and atleast one spanning tree. **(10+10)**
22. (a) Prove that a graph with atleast one is 2 −chromatic iff it has no cycle of odd length.
    (b) Show that an $n-$ vertex graph is a tree iff its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda-1)^{n-1}$. **(10+10)**

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