

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**U.G. DEGREE EXAMINATION – STATISTICS****FIRST SEMESTER – APRIL 2022****16/17/18UMT1AL02 – MATHEMATICS FOR STATISTICS - I**

Date: 27-06-2022

Dept. No.

Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART A**Answer ALL the questions:****(10 X 2 = 20)**

1. Define scalar matrix.

2. Find the conjugate of the matrix $A = \begin{pmatrix} 1+i & 2+3i & 2 \\ 3-4i & 4+5i & 1 \\ 5 & 3 & 3-i \end{pmatrix}$.

3. Define eigen vectors.

4. State Cayley Hamilton theorem.

5. If $y = (2x^2 + 4)^3$, find $\frac{dy}{dx}$.6. Differentiate x^4 with regard to $\log(x + 2)$.7. Prove that the function $f(x) = x^3 - 3x^2 + 6$ is positive for all value of $x \geq 2$.8. Find the first order partial differential coefficients of $u = \log(ax + by)$.9. Evaluate $\int \left(4x^3 - 2x + \frac{3}{x^3}\right) dx$.

10. State any two properties of definite integrals.

PART B**Answer any FIVE questions:****(5 X 8 = 40)**11. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.12. Find the inverse of the matrix $B = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$.13. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.14. Find the differential coefficient of $y = \frac{(x^2-1)^{4/5} (3x+5)^{2/7} e^{3x}}{(x-9)^{1/2}(2x-7)^4}$ with respect to x .15. Show that for $x > 0$, $x - \frac{1}{2}x^2 < \log(1+x) < x$.16. Verify Euler's theorem for the function $u = x^3 + y^3 + z^3 + 3xyz$.17. Evaluate $\int \frac{2x+3}{x^2+x+1} dx$.18. Prove that $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}}}{(\sin x)^{\frac{3}{2}} + (\cos x)^{\frac{3}{2}}} dx = \frac{\pi}{4}$.

PART C

Answer any TWO questions:

(2 X 20 = 40)

19. (a) Show that the matrix $B = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is orthogonal.

(b) Solve $x + y + z + 1 = 0$; $x + 2y + 3z + 4 = 0$; $x + 3y + 4z + 6 = 0$ using Cramer's rule. (10+10)

20. (a) Find the characteristic roots and associated characteristic vectors of the matrix

$$A = \begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}.$$

(b) For what values of x is the curve $y = 3x^2 - 2x^3$ concave upwards and when is it convex upwards? (15+5)

21. Find the maximum or minimum values of the function $u = 2(x^2 - y^2) - x^4 + y^4$. (20)

22. (a) Evaluate $\int x^2 e^x dx$.

(b). Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)} dx$. (5+15)

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