LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2022**

PMT 4505 – CLASSICAL MECHANICS

Date: 22-06-2022 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer ALL the questions

1. (a) State and prove D'Alembert's principle. (5)

(OR)

(b) Find the Lagrange's equation for motion of a particle in cartesian coordinates. (5)

(c) (i) A bead slides on a wire in the shape of a cycloid described by equations $x = a(\theta - sin\theta)$, $y = a(1 + cos\theta)$ where $0 \leq \theta \leq 2\pi$. Find the Lagrangian function and Lagrange's equation of motion.

(ii) Write down the Lagrangian equation if the Lagrangian has the form $L = -(1 - \dot{q}^2)^{1/2}$ (10+5)

(OR)

(d) State and prove the conservation theorem for the total energy for a system of particles. (15)

2. (a) If the system is conservative and the coordinate transformation is independent of time then prove that the Hamiltonian function equals the total energy of the system. (5)

(OR)

(b) A simple pendulum hangs from the ceiling of an elevator which is moving down with a constant acceleration f. Obtain the Hamiltonian function for the simple pendulum. (5)

(c) (i) If the component of the applied torque along the axis of rotation vanishes then prove that the component of total angular momentum along the axis of rotation is conserved.

(ii) Derive Hamilton's canonical equation of motion. (10+5)

(OR)

(d) State Hamilton's principle and derive it from D'Alembert's principle. (15)

3. (a) Prove that Lagrange's bracket is invariant under canonical transformation and anti-commutative.

. (5)

(OR)

(b) Using Poisson bracket, show that the transformation $Q = (e^{-2q} - p^2)^{\frac{1}{2}}$, $P = cos^{-1}(pe^q)$ is canonical. (5)

(c) State and prove Principle of least action. (15)

(OR)

(d) Prove that the integral $J_1 = \iint_S \sum_i dq_i \, dp_i$ taken over an arbitrary two dimensional surface s of the $2n$ dimensional phase space is invariant under canonical transformation. (15)

4. (a) Derive the transformation equation for infinitesimal contact transformation in terms of Poisson bracket. (5)

(OR)

(b) Derive equations of motion in Poisson bracket. (5)

(c) (i) If q_j is cyclic then prove that the corresponding generalized momentum is constant.

(ii) Prove that the Hamilton Principle function $S = \int Ldt +$ constant, where L is the Lagrangian. (7+8)

(OR)

(d) State and prove Liouville's theorem. (15)

5. (a) Derive Hamilton Jacobi equation for Hamilton's characteristic function. (5)

(OR)

(b) Write a brief note on the types of periodic function. (5)

(c) Find the solution for Simple Harmonic Oscillator problem by Hamilton Jacobi method. (15)

(OR)

(d) Discuss the separation of variables in Hamilton Jacobi equation. (15)

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