LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2022**

UMT 1501 – ALGEBRA

(19 & 20 BATCH ONLY)

Date: 15-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL questions. $(\mathbf{10 \times 2 = 20})$

1. Form a quadratic equation, given that $- - 2 + \sqrt{-7}$ is a root.
2. Define reciprocal equation.
3. Find the number of real roots of the equation $x^3 + 18x - 6 = 0$.
4. Find the interval in which a root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ lies.
5. State Cayley Hamilton theorem.
6. Use Binomial Theorem to find the 3rd power of 11.
7. What is the Characteristic equation of a matrix?
8. Define similar matrices.
9. Find the number of integers less than and prime to 720.
10. Find the number of divisors of 360.

PART - B

Answer any FIVE questions: $(\mathbf{5 \times 8 = 40})$

11. Show that the sum of the eleventh powers of the roots of $x^7 + 5x^4 + 1 = 0$ is zero.
12. Diminish the roots of the equation $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2 and write the transformed equation.
13. State and prove Fermat's theorem.
14. Solve $\mathrm{x}^3 - 27\mathrm{x} + 54 = 0$ by Cardon's method.
15. Find the sum of the series to infinity using binomial series expansion $\frac{15}{16} + \frac{15.21}{16.24} + \frac{15.21.27}{16.25.32} + \cdots$.
16. Find the characteristic equation of the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and hence find its inverse
17. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.
18. Show that $3^{2n+1} + 2^{n+2}$ is divisible by 7.

PART– C

Answer Any TWO Questions. **(2 X 20 = 40)**

19. a) Prove that the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ are in arithmetic progression, if $2p^3 - 9pq + 27r = 0$.

b.) Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0.$ (8+12)

20. Using Horner's Method find the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2 correct to three decimal places.

21. a.) Show that $\log\sqrt{12} = 1 + \left(\frac{1}{2}+\frac{1}{3}\right).\frac{1}{4} + \left(\frac{1}{4}+\frac{1}{5}\right).\frac{1}{4^2} + \left(\frac{1}{6}+\frac{1}{7}\right).\frac{1}{4^3} + \cdots.$

b.) Sum the infinite series $\frac{3}{2!} + \frac{5}{4!} + \frac{7}{61} + \cdots.$ (10+10)

22. Determine the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

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