# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2022**

**UMT 2501 – ANALYTICAL GEOMETRY**

Date: 16-06-2022 Dept. No. Max. : 50 Marks

Time: 01:00-04:00

**Part A (Answer ALL questions)** **( 10 x 2 = 20 )**

1. Write the condition for a line $y = mx + c$ to be a tangent to the parabola $y^2 = 4ax$.
2. Define Polar Co-ordinates.
3. Find the point to the pole of the line $Ax + By + C = 0$ with respect to the parabola $y^2 = 4ax$.
4. If the asymptotes of the hyperbola is $lx + my + n = 0$ and $l_1x + m_1y + n_1 = 0$, then what is the equation of the hyperbola?
5. Write the centre and radius for the general equation of a sphere.
6. Define great circle.
7. Show that the points $(5, 3, -2), (3, 2, 1), (-1, 0, 7)$ are collinear.
8. Prove that the line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-4}{-1}$ is parallel to the plane $x - 2y - 4z + 7 = 0$.
9. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the given point $(1, 2, 3)$.
10. Define enveloping cylinder.

**Part B (Answer any FIVE questions)** **( 5 x 8 = 40 )**

11. Identify the locus of the poles of chords of a parabola subtending a right angle at the vertex.
12. The chords of the parabola are drawn through a fixed point. Show that the locus of the middle point is another parabola.
13. Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.
14. If $e_1$ and $e_2$ are the eccentricities of a hyperbola and its conjugate, the prove that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$.
15. Find the equation of the plane through the intersection of two planes $x + y + z = 1$, $2x + 3y + 4z - 7 = 0$ and perpendicular to the plane $x - 5y + 3z = 5$.
16. Find the equation of the plane passing through two points $(-1, 3, 2)$ and perpendicular to the two planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$
17. A spehere of radius $k$ passes through the origin and meets the axis in $A, B$ and $C$. Prove that the centroid of the triangle $ABC$ lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$.
18. Prove that the circles $x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0$, $5y + 6z + 1 = 0$ and the $x^2 + y^2 + z^2 - 3x - 4y + 5z - 6 = 0$, $x + 2y - 7z = 0$ lie on the same sphere and find its equation.

**Part C (Answer any TWO question)** **( 2 x 20 = 40 )**

19. (a) If P and Q are the eccentricities of the conjugate diameters of the ellipse, then prove the following:
    a. The locus of the middle point of PQ is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$.
    b. The locus of the foot of the perpendicular on PQ from the centre of the ellipse is $a^2x^2+b^2y^2=2(x^2+y^2)^2$.

    (b) Prove that the tangent to a rectangular hyperbola terminated by its asymptotes is bisected at the point of contact and encloses a triangle of constant area. **(10+10)**

20. (a) Show that the locus of the intersection of tangents to $y^2=4ax$ which intersect a constant length '$d$' on the directrix is $(y^2=4ax)(x+a)^2=d^2x^2$.

    (b) Find the angle between the lines joining the points $(3,1,-2),(4,0,-4)$ and $(4,-3,3)$, $(6,-2,2)$. **(10+10)**

21. (a) Trace the conic $\frac{12}{r}=4+\sqrt{3}\cos\theta+3\sin\theta$.

    (b) A line makes angels $\alpha,\beta,\gamma,\delta$ with the four diagonals of a cube. Prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$. **(10+10)**

22. (a) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0$, $x+2y+3z=8$ and touches the plane $4x+3y=25$.

    (b) Show that the equation of the right circular cone whose vertex is $O$ and axis $OZ$ and the semi vertical angle $\alpha$ is $x^2+y^2=z^2\tan^2\alpha$. **(10+10)**

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