LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2022**

UMT 2502 – TRIGONOMETRY, FOURIER SERIES AND VECTOR ANALYSIS

Date: 18-06-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

PART – A

Answer ALL the Questions: **(10 × 2 = 20)**

1. Express $\sin x$ in terms of powers of x.
2. Give the expression for $x^n + \frac{1}{x^n}$ when $x = \cos y + i \sin y$.
3. Write down $cosh^{-1}x$ in terms of logarithmic function.
4. Determine $\text{Log}_e(-5)$.
5. Expand b_n in the Fourier series expansion for $f(x)$ in the interval $0 \le x \le 2\pi$.
6. What are the conditions for a function f to have a Fourier series expansion?
7. Define derivative of a vector-valued function of a scalar variable t.
8. Give a geometrical interpretation for gradient of real-valued function φ.
9. If S is any closed surface, find $\iint_S \left(ax\vec{i} + by\vec{j} + cz\vec{k}\right).\vec{n}\, dS$.
10. Recall and state Green's theorem.

PART – B

Answer any FIVE of the following: **(5 × 8 = 40)**

11. Express $\cos 8x$ in terms of $\sin x$.
12. Find $\lim_{x\to 0} \frac{\tan x + \sec x - 1}{\tan x - \sec x + 1}$.
13. If $\cos(x + iy) = cos\theta + isin\theta$, prove that $cos2x + cosh2y = 2$.
14. If $\tan(x + iy) = u + iv$, check whether $\frac{u}{v} = \frac{sin2x}{sinh2y}$.
15. Determine a cosine series for the function $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$.
16. Estimate the value of b such that the vector $\vec{f} = (bxy - z^3)\vec{i} + (b-2)x^2\vec{j} + (1-b)xz^2\vec{k}$ is irrotational.
17. Prove that $div(r^n\vec{r}) = (n+3)r^n$. Deduce that $r^n\vec{r}$ is solenoidal if and only if $n = -3$.
18. Evaluate $\int_C (x^2 + y^2)dx - 2xydy$ where C is the rectangle in the xy-plane bounded by $y = 0$, $y = b$, $x = 0$ and $x = a$ using Green's theorem.

PART – C

Answer any TWO of the following: **(2 × 20 = 40)**

19. a) Establish the formula $16\sin^5 x = \sin 5x - 5\sin 3x + 10\sin x$.

    b) Write the real and imaginary parts of $\tan^{-1}(x+iy)$.

20. Express x^2 as $\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $(-\pi \le x \le \pi)$. Deduce that

    a) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12}$.

    b) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$.

21. a) Predict the angle between the surfaces $x^2+y^2+z^2 = 29$ and $x^2+y^2+z^2+4x-6y-8z-47=0$ at $(4,-3,2)$.

    b) Determine the equation of the tangent plane and normal line to the surface $xyz = 4$ at the point $(1,2,2)$.

22. Check Gauss Divergence theorem for the vector function $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$, $z = 2$.

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