LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2022**

UMT 2502 – TRIGONOMETRY, FOURIER SERIES AND VECTOR ANALYSIS

(21 BATCH ONLY)

Date: 18-06-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION – A		
Answer ALL the Questions			
1.	**Answer the following:**		**(5 x 1 = 5)**
a)	Expand $\sin nx$.	K1	CO1
b)	Write an expression for $cosh^{-1}x$.	K1	CO1
c)	State the Fourier series expansion for $f(x)$ in the interval $0 \leq x \leq 2\pi$.	K1	CO1
d)	Give an expression that represents a normal to the surface $\varphi(x, y, z) = c$.	K1	CO1
e)	Write the statement of Stoke's theorem.	K1	CO1
2.	**Fill in the blanks**		**(5 x 1 = 5)**
a)	In terms of x, the series expansion of $\cos x$ is ______________.	K1	CO1
b)	$sinh(x + y) =$ ______________.	K1	CO1
c)	Fourier coefficient b_n for $f(x) = x$ in $-\pi < x < \pi$ is ______________.	K1	CO1
d)	The operator ∇^2 is called ______________.	K1	CO1
e)	Suppose a particle acted upon by a force $\vec{F}$ describes an arc C on moving from a point $\vec{r}$ to a point $\vec{r} + \Delta\vec{r}$ then the work done is ______________.	K1	CO1
3.	**Choose the correct answer for the following**		**(5 x 1 = 5)**
a)	In the expansion of $\left(\cos x + \cos\frac{1}{x}\right)^n$, for n being even, the term independent of x is (i) $nC_{\frac{n}{2}}$ (ii) $nC_{\frac{n}{2}+1}$ (iii) $nC_{\frac{n-1}{2}}$ (iv) $nC_{\frac{n+1}{2}}$	K2	CO1
b)	$tanh^{-1}x =$ (i) $\frac{1}{2}\log_e\left(\frac{1+x}{1-x}\right)$ (ii) $\frac{1}{2}\log_e\left(\frac{1-x}{1+x}\right)$ (iii) $\frac{1}{2}\log_e\left(\frac{x-1}{x+1}\right)$ (iv) $\log_e\left(\frac{1+x}{x}\right)$	K2	CO1
c)	Fourier coefficient a_0 in the half range cosine series of the function $f(x) = \pi x - x^2$ in $0 \leq x \leq \pi$ is (i) $\frac{\pi}{3}$ (ii) $\frac{\pi^2}{3}$ (iii) $\frac{\pi}{6}$ (iv) $\frac{\pi^3}{3}$	K2	CO1
d)	A vector $\vec{f}$ is called a harmonic vector if (i) $\nabla\vec{f} = 0$ (ii) $\nabla^2\vec{f} = 0$ (iii) $\nabla\vec{f} = 1$ (iv) $\nabla^2\vec{f} = 1$	K2	CO1
e)	If R is a closed region of the xy - plane bounded by a simple closed curve C and M, N are continuous functions of x and y having continuous first order partial derivatives in R, then $\int_C Mdx + Ndy$ equals (i) $\iint_R \left(\frac{\partial N}{\partial x} + \frac{\partial M}{\partial y}\right) dxdy$ (ii) $\iint_R \left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}\right) dxdy$ (iii) $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy$ (iv) $\iint_R \left(\frac{\partial N}{\partial y} + \frac{\partial M}{\partial x}\right) dxdy$		
4.	**Say TRUE or FALSE**		**(5 x 1 = 5)**
a)	The series expansion of $\sin \pi$ is $1 + \frac{\pi^2}{2!} + \frac{\pi^4}{4!} + \cdots$.	K2	CO1

b)	$Log_e(-5) = log5 + i(2n+1)\pi$.	K2	CO1
c)	Coefficient a_0 in the Fourier series expansion for the function $f(x) = e^x$ defined in $-\pi < x < \pi$ is $\frac{2}{\pi} sinh\pi$.	K2	CO1
d)	The gradient of a scalar-valued function is a vector.	K2	CO1
e)	If S is a closed surface, then $\iint_S \vec{r}\,.\vec{n}\,dS = 3V$ where V is the volume enclosed by S.	K2	CO1
SECTION – B			
Answer any TWO of the following in 100 words		**(2 x 10 = 20)**	
5.	Find $\lim_{x\to 0} \frac{\tan x + \sec x - 1}{\tan x - \sec x + 1}$.	K3	CO2
6.	If $\tan(x+iy) = u + iv$, check whether $\frac{u}{v} = \frac{sin2x}{sinh2y}$.	K3	CO2
7.	Derive a sine series for the function $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < x < \pi \end{cases}$.	K3	CO2
8.	If $\varphi = x^2 z + e^{y/x}$ and $\psi = 2z^2 y - xy^2$, determine $\nabla(\varphi + \psi)$ and $\nabla(\varphi\psi)$ at (1,0,2).	K3	CO2
SECTION C			
Answer any TWO of the following in 100 words		**(2 x 10 = 20)**	
9.	Evaluate $\lim_{x\to 0} \frac{\sin 2x - 2\sin x}{x^3}$.	K4	CO3
10.	If $\cos(x+iy) = cos\theta + isin\theta$, show that $cos2x + cosh2y = 2$.	K4	CO3
11.	Determine a cosine series for the function $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$.	K4	CO3
12.	Check Green's theorem for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region R enclosed by $y = x^2$ and $x = y^2$.	K4	CO3
SECTION – D			
Answer any ONE of the following in 250 words		**(1 x 20 = 20)**	
13.	(a) Show that $2^6 \cos^7 x = \cos 7x + 7\cos 5x + 21\cos 3x + 35\cos x$. **(10 Marks)** (b) Construct a series of hyperbolic cosines of multiples of θ for $sinh^6\theta$ and $cosh^6\theta$. **(10 Marks)**	K5	CO4
14.	(a) Express the function $f(x) = \begin{cases} x+1, & 0 < x < \pi \\ x-1, & -\pi < x < 0 \end{cases}$ in $(-\pi \le x \le \pi)$ as a Fourier series. **(10 Marks)** (b) Prove that $\vec{f} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is both irrotational and solenoidal. **(10 Marks)**	K5	CO4
SECTION – E			
Answer any ONE of the following in 250 words		**(1 x 20 = 20)**	
15.	Develop a Fourier series for x^2 as $\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $(-\pi \le x \le \pi)$. Deduce that (a) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12}$. (b) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$.	K6	CO5
16.	Given a vector-valued function $\vec{F} = (x^3 - yz)\vec{i} - 2x^2 y\vec{j} + 2\vec{k}$, verify Gauss Divergence theorem for $\vec{F}$ over the cube bounded by $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$, $z = 2$.	K6	CO5

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