LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

FOURTH SEMESTER – **APRIL 2022**

UMT 4402 – MATHEMATICS FOR PHYSICS - II

Date: 27-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

Part A

Answer ALL the questions **(10 × 2 = 20)**

1. Find the constant a_0 of the Fourier series for the function $f(x) = x$ in $0 < x < 2\pi$.
2. Obtain the sine series for unity in $(0, \pi)$.
3. What is Clairut's equations?
4. What is a particular solution of differential equation?
5. Define Linear differential equation.
6. If the roots are real and distinct that is α and β, then what is complementary function?
7. Find $L[t^2 e^{-3t}]$.
8. Find $L^{-1}[\frac{s}{(s-b)^2+a^2}]$.
9. Find the directional derivative of $\varphi = x^2yz + 4xz^2$ at (1,-2,-1) in the direction of $2\vec{i} - \vec{j} - 2\vec{k}$.
10. Find 'a' such that $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.

Part B

Answer any FIVE questions **(5 × 8 = 40)**

11. Find the Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$.
12. Obtain the Fourier expansion of $x\, sinx$ as a cosine series in $(0, \pi)$.
13. Solve $(D^4 - 1)y = \cos\ x \cosh x$.
14. Solve the differential equation $(1 + xy)ydx + (1 - xy)xdy = 0$.
15. Solve the differential equation $\frac{y+x-2}{y-x-4}$.
16. Find (i) $L[t^2e^t sint]$, (ii) $L[\frac{1-cost}{t}]$.
17. Find (i) $L^{-1}[\frac{5s^2-15s-11}{(s+1)(s-2)^3}]$, (ii) $L^{-1}[\frac{1}{s(s^2-2s+5)}]$.
18. Using Green's theorem, evaluate $\int_C \{(3x - 8y^2)dx + (4y - 6xy)dy\}$ where C is the boundary of the region given by $x = 0, y = 0, x + y = 1$.

Part C

Answer any TWO question **(2 × 20 = 40)**

19. (a) Find the Fourier series expansion of the periodic function $f(x)$ of the period 4 defined by

$f(x) = \begin{cases} 1 + x & -2 \le x \le 0 \\ 1 - x & 0 \le x \le 2 \end{cases}$. Hence deduce that $\sum_1^\infty \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

(b) Find a Fourier series to represent x^2 in the interval $(-l, l)$. **(12+8)**

20. (a) Solve the differential equation $x\frac{dy}{dx} + y = x^3y^6$.

(b) Solve $(D^2 - 6D + 25)y = e^{2x} + \sin\ x + x$. **(10+10)**

21. (a) Solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$ given that $y = 0, \frac{dy}{dt} = 2$ when $t = 0$.

(b) Using convolution theorem find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$. **(15+5)**

22. (a) Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

(b) If $\vec{F} = x^2y\vec{i} + y^2z\vec{j} + z^2x\vec{k}$, then find $curl\ (curl\vec{F})$. **(15+5)**

########