

# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



**B.A. DEGREE EXAMINATION – ECONOMICS**

**FOURTH SEMESTER – APRIL 2022**

**UMT 4404 – ADVANCED MATHEMATICS FOR ECONOMICS**

Date: 27-06-2022

Dept. No.

Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

## PART-A

**Answer all the questions**

**(10 x 2 = 20 Marks)**

1. What output will maximize total revenue if  $TR = 250q - 2q^2$  ?
2. Define reorder costs & storage costs.
3. When will you use substitution method.?
4. Write the Lagrange equation for constrained minimization with two variables.
5. What are the assumptions in the cobweb model?
6. Write the condition for the stability and unstability market in the cobweb model.
7. Write the formula that is used to find the final value of any variable growing continuously at a known annual rate from a given original value.
8. Population in a developing country is growing continuously at an annual rate of 3%. If the population is now 4.5 million, what will it be in 15 year's time?
9. Write the solution for the non-homogeneous differential equations.
10. Find the particular solution to the differential equation  $\frac{dy}{dt} = 6y + 27$ .

## PART-B

**Answer any 5 Questions**

**(5 x 8 = 40 Marks)**

11. A firm uses 200,000 units of a components in a year with demand evenly spread over the year. In addition to the purchase price, each order placed for a batch of components costs Rs 80. Each unit in stock over a year costs Rs. 8. What is the optimum order size.
12. A firm faces the production function  $Q = 6K^{0.4}L^{0.5}$ . If it can buy input K at Rs 32 a unit and input L at Rs 8 a unit, what combination of L and K should it use to maximize production if it is constrained by a fixed budget of Rs 36000?.
13. A firm faces the production function  $Q = 50K^{0.5}L^{0.2}R^{0.2}$ . It has a budget of Rs 24,000 and can buy K,L and R at Rs. 80, Rs.12 and Rs. 10 respectively per unit. Using lagrange method find the combination of inputs that will maximize its output.
14. Write the procedure to solve the cobweb difference equations and discuss about the stability and unstability conditions.
15. In a basic Keynesian macoeconomic model it is assumed that initially  $Y_t = C_t + I_t$ , where  $I_t = 134$  is exogenously determined and  $C_t = 40 + 0.06Y_{t-1}$ . The level of investment  $I_t$  then falls to 110 and remains at this level each time period. Trace out the pattern of adjustment to the new equilibrium value of Y, assuming that the model was initially in equilibrium.
16. A renewable natural resource R will allow an estimated maximum consumption rate of 200 million units per annum. Current annual usage is 65 million units. If the annual level of usage grows continuously at an annual rate of 7.5%, will there be sufficient R to satisfy annual demand after (a) 5 years (b) 10 years (c) 15 years (d) 20 years.

17. For the differential equation  $\frac{dy}{dx} = 0.4y - 80$  with initial value  $Y_0 = 180$
- Derive the definite solution
  - Use the solution to predict the value of  $y$  when  $t$  is 5
18. If the demand and supply functions in a competitive market are  $Q_d = 35 - 0.5P$  and  $Q_s = -4 + 0.8P$ , and the rate of adjustment of price when the market is out of equilibrium is  $\frac{dP}{dt} = 0.25(Q_d - Q_s)$ .  
Derive and solve the relevant differential equation to get a function for  $P$  in terms of  $t$  given that price is 37 in time period 0. Comment on the stability of the market.

**PART-C**

**( 2 x 20 = 40 Marks)**

**Answer any 2 Questions**

19. a). A monopoly faces the total revenue schedule  $TR = 300q - 2q^2$  and the total cost schedule  $TC = 12q^3 - 44q^2 + 60q + 30$ . Are there two output levels at which  $MC=MR$  ? What is the profit maximizing output? ( 10 marks )
- b) Find whether any stationary point exist for the following functions for positive values of  $q$ , and say whether or not the stationary points are at the minimum values of the functions.
- $MC = 30 + 0.4q^2$
  - $MC = 8.25q$  ( 10 marks)
- 20.a) A firm can buy two inputs  $K$  and  $L$  at Rs 18 per unit and Rs. 8 per unit respectively and faces the production function  $Q = 24K^{0.6}L^{0.3}$ . What is the maximum output it can produce for a budget of Rs 50,000? ( 10 marks)
- b). A firm has a budget of Rs 300 to spend on the three inputs  $x$ ,  $y$  and  $z$  whose prices per unit are Rs.4, Rs.1 and Rs. 6 respectively. What combination of  $x$ ,  $y$ ,  $z$  should it employ to maximize output if it faces the production function  $Q = 24x^{0.3}y^{0.2}z^{0/3}$  ? ( 10 marks)
- 21 a). In an agricultural market where the assumptions of the cobweb model apply, the demand and supply schedules are  $Q_t^d = 3450 - 6P_t$  and  $Q_t^s = -729 + 4.5P_{t-1}$ . Use difference equation to predict what price will be in the tenth period after an unexpected drop in quantity to 354, assuming that the market was previously in long-run equilibrium. ( 12 marks)
- b). World reserves of minerals  $M$  are observed to have declined from 830 millions tonnes to 657 million tons over the last 25 years. Assuming this decline has been continuous, calculate the annual rate of decline and then predict what reserves will be left in 10 years. ( 8 marks)
- 22 a). Derive the solution of non-homogeneous differential equation  $\frac{dy}{dx} = by + c$ . ( 8 marks)
- b). In a macroeconomic model  $c = 200 + 0.75Y$ ,  $E = C + I$  and  $I = 80$ ,  
 $\frac{dY}{dt} = 0.8(E - Y)$  . If  $Y_0 = 1200$ , derive a function for  $Y$  in terms of  $t$  and comment on the stability of this model. ( 12 Marks)

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