LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2022**

UMT 4601 – COMBINATORICS

Date: 23-06-2022 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL Questions: **(10 × 2 = 20)**

1. Find $f\ (n, k)$ where $n = 4$ and $k = 2$. [using recurrence relation].
2. How many different binary sequences of length 10 containing exactly 5 zeros?
3. Tom has 75 books but enough room on his book shelf for only 20. In how many ways can he fill his shelf?
4. A binary sequence of length n is a string of n digits each of which is 0 or 1. How many such sequences are there?
5. Construct 2 different $5\ \times\ 5$ Latin square which have the same first row
6. 10 people meet and form 5 pairs. How many ways their pairs can obtain?
7. When a path or trail is said to be closed?
8. Write any one possible derangements of 1 2 3 4 5.
9. When a board is said to have a forbidden position?
10. Find the rook polynomial of n—non intersecting 2×2 blocks.

PART – B

Answer any FIVE Questions. **(5 × 8 = 40)**

11. Suppose that $t(n, n-1) = 1$ and $(n-k-1)\, t(n,k) = k(n-1)\, t(n, k+1)$ for each $k < n-1$. Show that $t(n,k) = \frac{(n-1)^{n-k-1}(n-2)!}{(k-1)!(n-k-1)!}$.
12. If a football league of n teams, each team plays each other twice. The number of games played is therefore $2C$, where C is the number of ways choosing two objects from n given objects. Prove that $C = (n-1) + (n-2) + \cdots \ldots + 2 + 1 = \frac{n(n-1)}{2}$ and deduce the number of games played in a league of 22 teams.
13. (a) In how many ways can a 5-letter word be formed from an alphabet of 26 letters.
    i. if reputation are allowed?
    ii. if reputation are not allowed?

    (b) Explain and derive unordered selection. **(4+4)**
14. (a) How many different necklaces can be designed from n colour, using one bead of each colour?

    (b) Show that $\binom{7}{4} = \binom{6}{3} + \binom{6}{4}$. **(4+4)**
15. Find the optimal assignment to the following problem:

	Man				
Job		A	B	C	D
	A	6	8	2	7
	B	5	8	13	9
	C	2	7	8	9
	D	4	11	7	10

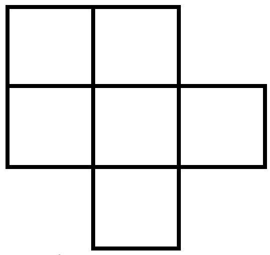
16. A n digit integer sequence are to be formed using only the integers $0,1,2,3, \ldots ..$
    a. How many n digit sequences are there?
    b. How many n-digit sequences have an odd number of zeros?

17. Find the general formula for U_n, the number of different rooted trees.
18. Given a chessboard C, choose any square of C and let D denote the board obtained by deleting from C every square in the same row or column at the chosen square (including the chosen square itself). Let E denote the board obtained from C by deleting only the chosen square. Then prove that $R(x, C) = xR(x, D) + R(x, E)$.

PART – C

Answer any TWO Questions. **(2 × 20 = 40)**

19. (a) Suppose that each of k—indistinguishable golf balls have to be colored with any one of n colours using binomial theorem (generating function) approach. Find out how many different colouring are possible and hence deduce the case $k = 4$ and $n = 9$.
    (b) Explain ordered selection and evaluate the following: (i) p(7,4), (ii) p(9,5). **(12+8)**
20. (a) Let n be a positive integer. Show that if $(1+x)^n$ is expanded as a sum of powers on n, the coefficient of x^r is $\binom{n}{r}$.
    (b) Find a_n if $a_n = 4a_{n-1} + 4a_{n-2} - 16a_{n-3}$, $a_1 = 8$, $a_2 = 4$, $a_3 = 24$. **(12+8)**
21. (a) Find the value of k_2 given, $\left(\frac{\sqrt{5}+1}{2\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right) + k_2\left(\frac{1-\sqrt{5}}{2}\right) = 1$.
    (b) State and prove Marriage Theorem. **(8+12)**
22. (a) Find the rook polynomial of the board



    (b) Derive $a_n = n!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + (-1)^n \frac{1}{n!}\right\}$ by using inclusion and exclusion principle.

(10+10)

@@@@@@@