LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2022**

UMT 5503 – DISCRETE MATHEMATICS

Date: 17-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL questions **(10x2= 20)**

1. Show that $P \wedge Q$ implies $(\neg P \rightarrow Q)$.
2. Define monoid with an example.
3. Prove that in a lattice if $a \leq b$ then $a \oplus b = b$.
4. How do you construct the direct product of two Boolean algebras?
5. Write the following statement in symbolic form, 'Moscow is neither a country nor a state'.
6. What is an idempotent element?
7. When do you say an element to be join-irreducible?
8. State the rules of inference.
9. Discuss the conditions for a Boolean expression to be symmetric.
10. Illustrate substitution instance with an example.

PART – B

Answer any FIVE questions **(5 x 8 = 40)**

11. Prove that the set of all functions from X to X forms a semigroup under the operation of composition of mappings. Also verify whether it forms a monoid.
12. Construct the truth table of the following statements:

    (a) $(Q \wedge (P \rightarrow Q)) \rightarrow P$

    (b) $\neg(P \wedge Q) \rightleftarrows (\neg P \vee Q)$.
13. (a) Show that $P(x) \wedge (x)Q(x) \Rightarrow (\exists x)(P(x) \wedge Q(x))$.

    (b) Prove that the conclusion $R \vee S$ follows from the premises $(C \vee D) \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$ and $(A \wedge \neg B) \rightarrow (R \vee S)$ using equivalence laws.
14. Show that in a complemented distributive lattice $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$.
15. Prove that the quotient set $(S/R, \oplus)$ is a semigroup, where R is congruence relation defined on a semigroup (S,*). Also verify whether there exists a natural homomorphism from (S,*) onto $(S/R, \oplus)$.
16. Show that $P \rightarrow Q$, $P \rightarrow R$, $Q \rightarrow \neg R$ and P are inconsistent.
17. State and prove isotonicity property of Lattices.
18. Reduce the following expressions where + represents the operation in Boolean algebra.

    i. $ab + abc + abc' + a'bc$.

ii. $a(a+c) = aa + ac$.

PART – C

<u>Answer any TWO questions</u> **(2 x 20 = 40)**

19. (a) Express the following Boolean expressions in an equivalent sum of the product of canonical forms in three variables $x_1, x_2\ and\ x_3$ $(i)\ x_1 * x_2$. $(ii)\ x_1 \oplus x_2$. $(iii) (x_1 \oplus x_2)' * x_3$.

    (b) Obtain the principal disjunctive and conjunctive normal forms of

    (Q → P) ∧ (˥P ∧ Q). **(10 + 10)**

20. (a) What is a Boolean algebra? List down its various properties.

    (b) Show that the set $N = \{0,1,2,..\}$ is a semigroup under the operation defined by

    $x * y = max\{x, y\}$. Also check whether it forms a monoid. **(10 + 10)**

21. (a) Let X be a set containing n elements, let X^* denote the free semigroup generated by X, and let $(S, \oplus)$ be any other semigroup generated by any n generators then show that there exists a homomorphism $g: X^* \to S$.

    (b) Using truth tables verify whether

    $\big(P \to (Q \to R)\big) \Leftrightarrow \big(P \to (˥Q \vee \mathrm{R})\big) \Leftrightarrow \big((P \wedge Q) \to R\big)$. **(10 + 10)**

22. (a) Prove that (S_{42}, D) the set of all divisors of 42 and D denotes the relation of division is a complemented lattice. Also evaluate the same for (S_n, D); $n =$ 12,8.

    (b) Verify using rules of inference whether $S \vee R$ is tautologically implied by

    $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$. **(10 + 10)**

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