LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2022**

UMT 2501 – ANALYTICAL GEOMETRY

Date: 16-06-2022 Dept. No. Max. : 50 Marks

Time: 01:00-04:00

SECTION A			
Answer ALL the Questions			
1.	**Answer the following** **(5 x 1 = 5 Marks)**		
a)	Define polar of a point with respect to the parabola $y^2 = 4ax$.	K1	CO1
b)	Define rectangular hyperbola.	K1	CO1
c)	Write the distance formula between the point $P(x_1, y_1, z_1)$ and the origin O.	K1	CO1
d)	Write the general equation of a sphere.	K1	CO1
e)	Define cone.	K1	CO1
2.	**Fill in the blanks** **(5 x 1 = 5 Marks)**		
a)	The condition for a pair of tangents to the parabola $y^2 = 4ax$ from the point (x_1, y_1) is ___________	^K1	CO1
b)	The equation of asymptotes of the hyperbola are ___________.	K1	CO1
c)	The direction cosines of the X-axis is ____________.	K1	CO1
d)	The length of the tangent from the point (x_1, y_1, z_1) to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ is _________________.	K1	CO1
e)	The fixed straight line in a right circular cone is called ________ of the cone.	K1	CO1
3.	**Choose the correct answer** **(5 x 1 = 5 Marks)**		
a)	The angle between the asymptotes is (i)$sec^{-1}(e)$ (ii) $2sec^{-1}(e)$ (iii) $tan^{-1}(e)$ (iv) $2tan^{-1}(e)$	K2	CO1
b)	The locus of the poles of chords of a parabola subtending a right angle at the vertex (i) $x + 4a = 0$(ii) y+$4a = 0$(iii) $x - 4a = 0$(iv) y$-4a = 0$	K2	CO1
c)	Let (l_1, m_1, n_1) and (l_2, m_2, n_2) be the direction cosines of two lines and θ be the angle between them. Then the equation $l_1 l_2 + m_1 m_2 + n_1 n_2 = 1$ is the condition for the lines to be (i) parallel (ii) perpendicular (iii) non-intersecting (iv) intersecting	K2	CO1
d)	The center of the sphere $x^2 + y^2 + z^2 + 2x - 4y + 6z + 5 = 0$. (i) $(-1,2,-3)$ (ii) $(-1,-2,3)$ (iii) $(1,-2,-3)$ (iv) $(1,2,3)$	K2	CO1
e)	Any plane which intersects the cylinder whose equation is of _______ degree is a conic (i) first (ii) second (iii) third (iv) fourth	K2	CO1
4.	**Say TRUE or FALSE** **(5 x 1 = 5 Marks)**		
a)	If p be the perpendicular on the tangent at P from the center of an ellipse, then $p \cdot CD = ab$.	K2	CO1
b)	The equation of the asymptotes does not differ from that of the hyperbola only in	K2	CO1

	the constant term.		
c)	Two straight non-intersecting lines in space are called non-skew lines.	K2	CO1
d)	The plane passing through the center of the sphere is called the great circle.	K2	CO1
e)	The locus of lines perpendicular to a given line which touches a given surface is called an enveloping cylinder.	K2	CO1

SECTION B

Answer any TWO of the following: (2 x 10 = 20 Marks)

5.	Derive the equation of pair of tangents to the parabola $y^2 = 4ax$ from the point (x_1, y_1).	K3	CO2
6.	Examine the tangent to a rectangular hyperbola terminated by its asymptotes, is bisected at the point of contact and encloses triangle of constant area.	K3	CO2
7.	Discover the equation of the plane passing through the points (2, 5, -3), (-2, -3, 5) and (5, 3, -3).	K3	CO2
8.	Examine the equation of the cone with vertex O and base curve, the conic in which the surface $ax^2 + by^2 + cz^2 = 1$ is cut by the plane $l_1x + m_1y + n_1z = p$.	K3	CO2

SECTION C

Answer any TWO of the following: (2 x 10 = 20 Marks)

9.	Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.	K4	CO3
10.	Find the equation of the cylinder whose generators are parallel to the z – axis and the guiding curve is $ax^2 + by^2 = cz$, $lx + my + nz = p$	K4	CO3
11.	Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$ and $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar. Find also the point of intersection and the plane through them.	K4	CO3
12.	Develop the equation of the sphere whose center is (6, -1, 2) and touches the plane $2x - y + 2z - 2 = 0$.	K4	CO3

SECTION D

Answer any ONE of the following: (1 x 20 = 20 Marks)

13.	a)	Define conjugate diameter of an ellipse and show that the focal distance of a point on an ellipse is equal to the square of the semi-diameter which is conjugate to the diameter through the point.	K5	CO4
	b)	If chords of a parabola are drawn through a fixed point, then show that the locus of the middle points is another parabola.	K5	CO4
14.	a)	Find the bisector of the acute angle between the planes $x + 2y + 2z - 3 = 0$ and $3x + 4y + 12z + 1 = 0$.	K5	CO4
	b)	Determine the equation of the enveloping cylinder of the surface $ax^2 + by^2 + cz^2 = 1$ having the generator parallel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.	K5	CO4

SECTION E

Answer any ONE of the following: (1 x 20 = 20 Marks)

15.	a)	Trace the curve $\frac{10}{r} = 3\cos\theta + 4\sin\theta + 5$.	K6	CO5
	b)	Compute the symmetrical form of the line $4x + 4y - 5z - 12 = 0 = 8x + 12y - 13z - 32$.	K6	CO5
16.	a)	Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - 6x + 2z + 1 = 0$ at the point (2, -2 , 1) and passes through the origin.	K6	CO5
	b)	Derive the equation of the sphere passing through the points (2, 3, 1), (5, -1, 2), (4, 3, -1) and (2, 5, 3).	K6	CO5

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