LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2023**

16/17/18UMT4MC01 – ABSTRACT ALGEBRA

Date: 02-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART A

Answer ALL the questions **(10 X 2 = 20)**

1. Define one-one mapping with an example.
2. If G is a group with order 7, then find all the possible orders of the elements of G.
3. Define a normal subgroup with an example.
4. Illustrate how two elements in a quotient group are operated.
5. Let G be the group of all integers with operation addition. Is the mapping $f: G \rightarrow G$ defined by $f(x) = x^2 + 1$ for all x in G, a group homomorphism?
6. Write the given permutation as product of disjoint cycles $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$.
7. Define a field with an example.
8. What is a division ring?
9. Prove that the kernel of a ring homomorphism is an ideal.
10. Define $J[i]$ where $i^2 = -1$.

PART B

Answer any FIVE questions **(5 X 8 = 40)**

11. If G is group of even order, then prove that it has an element $a \neq e$ satisfying $a^2 = e$.
12. If G is a group, show that for all $a \in G$, $Ha = \{x \in G \mid a \equiv x \bmod H\}$.
13. Show that a subgroup N of a group G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G.
14. If ϕ is a homomorphism of a group G into another group G' with kernel K, then prove that K is a normal subgroup of G.
15. If G is a group, then show that the set of all automorphisms $A(G)$ of G is a group.
16. Let H and K be finite subgroups of a group G, then show that $(HK) = \frac{o(H)o(K)}{o(H \cap K)}$.
17. If R is a commutative ring with unit element whose only ideals are (0) and R itself, then prove that R is a field.
18. Let R be a Euclidean ring. Show that any two elements a and b in R have a greatest common divisor d and $d = \lambda a + \mu b$ for μ, λ in R.

PART C

Answer Any TWO questions **(2 X 20 = 40)**

19. State and prove Lagrange's theorem with necessary lemmas. (20)
20. (a) State and prove the fundamental theorem of homomorphism for the groups.

    (b) State and prove Cayley's theorem. (10+10)
21. (a) If U is an ideal of a ring R show that R/U is also a ring and is a homomorphic image of R.

    (b) Prove that any field is an integral domain. (10+10)
22. (a) Show that J[i] is an Euclidean ring.

    (b) State and prove unique factorization theorem. (10+10)

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