LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2023**

PMT1MC01 – LINEAR ALGEBRA

Date: 29-04-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION A			
Answer ALL the questions			
1	**Answer the following** **(5 x 1 = 5)**		
a)	Write the minimal polynomial of $\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$	K1	CO1
b)	Define direct sum of a vector space.	K1	CO1
c)	Give an example for a nilpotent operator.	K1	CO1
d)	Define T-admissible subspace	K1	CO1
e)	Write the adjoint of an identity operator	K1	CO1
2	**Multiple Choice Questions** **(5 x 1 = 5)**		
a)	The eigen values of a nilpotent matrix of order 4 are a) 0, 0, 1, 1 b) 0, 0, 0, 0 c) 1, 1, 1 ,1 d) 1, 2, 3, 4	K2	CO1
b)	Similar matrices have a) Different characteristic polynomial. b) real eigen values c) Non negative eigen values d) Same characteristic polynomial	K2	CO1
c)	A linear operator has distinct eigen values then it is a) not diagonalizable b) diagonalizable c) nilpotent d) zero matrix	K2	CO1
d)	Let A be a matrix in rational form. Then each diagonal block of A is a) diagonal matrix b) triangular matrix c) companion matrix d) zero matrix	K2	CO1
e)	In R^2 $(\alpha\|\beta) = ax_1y_1 + bx_2y_2$ where $\alpha = (x_1, x_2)$, $\beta = (y_1, xy_2)$ is an inner product if a) $a = 0, b = -3$ b) $a = 2, b = 0$ c) $a = 2\ , b = 2$ d) For any real $a\ and\ b$.	K2	CO1
SECTION B			
	Answer any THREE of the following. **(3 x 10 = 30)**		
3	State and prove Cayley-Hamilton theorem.	K3	CO2
4	Let *V* be a finite-dimensional vector space over the field *F* and let *T* be a linear operator on *V*. Then prove that *T* is triangulable if and only if the minimal polynomial for *T* is a product of linear polynomials over *F*.	K3	CO2
5	Discuss about any four properties of nilpotent operators.	K3	CO2
6	If *U* is a linear operator on the finite dimensional space *W*, then show that *U* has a cyclic vector if and only if there is some ordered basis for *W* in which *U* is represented by the companion matrix of the minimal polynomial for *U*.	K3	CO2
7	Prove that an orthonormal set of non-zero vectors is linearly independent. Also construct an infinite orthonormal set.	K3	CO2
	SECTION C		
Answer any TWO of the following . **(2 x 12.5 = 25)**			
8	a)Let *T* be a linear operator on a finite-dimensional space *V* and let *c* be a scalar. Prove that the following are equivalent. i) c is a characteristic value of *T*.	K4	CO3

	ii) The operator $(T - cI)$ is singular (not invertible). iii) det $(T - cI) = 0$ b) Suppose $T\alpha = c\alpha$ and if f is a polynomial then show that $f(T)\alpha = f(c)\alpha$		
9	State and prove primary decomposition theorem.	K4	CO3
10	Let α be any non-zero vector in V and let p_α be the T-annihilator of α. Show that (i) The degree of p_α is equal to the dimension of the cyclic subspace $Z(\alpha; T)$. (ii) If the degree of p_α is k, then the vectors $\alpha, T\alpha, T^2\alpha, \ldots, T^{k-1}\alpha$ form a basis for $Z(\alpha; T)$. (iii) If U is the linear operator on $Z(\alpha; T)$ induced by T, then then the minimal polynomial for U is p_α.	K4	CO3
11	Let V be a finite-dimensional inner product space. If T and U are linear operators on V and c is a scalar then prove that (i) $(T+U)^* = T^* + U^*$; (ii) $(cT)^* = \bar{c}\, T^*$; (iii) $(TU)^* = U^*T^*$; (iv) $(T^*)^* = T$	K4	CO3
	SECTION D		
	Answer any ONE of the following . (1 x 15 = 15)		
12	Let V be a finite-dimensional inner product space, and f a linear functional on V. Then show that there exists a unique vector β in V such that $f(\alpha) = (\alpha\|\beta)$ for all α in V. Illustrate this theorem through an example on R^2.	K5	CO4
13	Let F be a field and let B be an n x n matrix over F. Then B is similar over the field F to one and only one matrix which is in rational form. If T is a nilpotent transform then discuss about each block in its rational form.	K5	CO4
	SECTION E		
	Answer any ONE of the following . (1 x 20 = 20)		
14	a) Let T be a linear operator on the finite-dimensional vector space V over the field F. Suppose that the minimal polynomial for T decomposes over F into a product of linear polynomials. Then show that there is a diagonalizable operator D on V and a nilpotent operator N on V such that (i)T = D + N . (ii)DN=ND.The diagonalizable operator D and the nilpotent operator N are uniquely determined by (i) and (ii) and each of them is a polynomial in T. b) Let V the space of all polynomials of degree less than or equal to 3. Let T be differential operator on V. Discuss about the properties of the matrix of T in the standard basis. Also write its Jordon form.	K6	CO5
15	Write abour the existence of cyclic decomposition theorem. Discuss about various possibilities of Jordon forms of A if it has characteristic polynomial $(x-3)^2(x-2)^4$ and has minimal polynomial $(x-3)(x-2)^2$.	K6	CO5

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